

## I. Abstract

The Democratic People's Republic of Korea (DPRK) has carried out five underground nuclear tests (UNE) since 2006 at the P'unggyeri test site. The explosions have been conducted certainly at closely located test sites. In this case, with an assumption of well coupled explosions, estimates on the relative depth and yield of the UNE can be attained by a spectral division of the two seismograms recorded at the same station because the source area's geological contribution to yield and attenuation effect along the seismic propagation path cancel out. We tried to estimate a relative depth and yield between DPRK's 3rd and 4th UNE using spectral division of Pn and Pg phases. The yield ratio, as a function of amplitude ratio at low frequency and depth ratio, is estimated along with another expression with corner frequency ratio and depth ratio. These two relationships give a solution of relative depth and yield between two events.

## II. Introduction

Democratic People's Republic of Korea (DPRK) conducted the fourth Underground Nuclear Test at their nuclear test in P'unggyeri on January 6, 2016 01:30:01 (UTC). Seismic signals from this event were recorded at seismic stations in the world as well as regional seismic stations in Republic of Korea. DPRK announced briefly after the underground nuclear explosion (UNE) that it was a successful hydrogen bomb test (<http://edition.cnn.com/2016/01/05/asia/north-korea-seismic-event/>). The announcement drew a great attention on the yield estimates of the explosion using seismic magnitude on the reason that a relatively bigger yield was expected from a hydrogen bomb compared to a regular nuclear bomb.

It is known that a fixed relationship between yield and seismic magnitude can not be held because for a single yield, the seismic magnitude can be various based on the mainly four parameters; rock type around source, attenuation characteristics of seismic energy from the source to seismic stations, depth of source burial and decoupling effect by a probable cavity [Mueller and Murphy, 1971; Denny and Johnson, 1991; Williams et al., 2012]. For DPRK's UNEs, the explosions have been performed almost at closely located test sites. In this case, with an assumption of well coupled explosions, relative estimates on depth and yield can be attained by a spectral division of the two seismograms recorded at the same station because the source area's geological contribution to yield and attenuation effect along the seismic propagation path can cancel out.

In this study, a relative depth and yield ratio calculated using spectral division of the Pn and Pg phases recorded from the DPRK's third and fourth UNE. The yield ratio between the two explosions can be expressed with two equations. The yield ratio of the two events detonated in the same geological medium is expressed as a function of amplitude ratio at low frequency and depth ratio. Another function expressed with corner frequency ratio and depth ratio also constrains the yield ratio as well. These two relationships give a solution of yield ratio with depth ratio between two events.

## III. Theory

Spectral ratio of amplitude at the static level and corner frequency ratio between two different explosions in the same geological medium can be used as input values for the solution of two simplified equations. The numerical solution of source depth and yield ratios can be attained by utilizing the two simplified equations.

### Amplitude ratio

In the same medium surrounding the explosive sources, yield ratio is expressed with a function of amplitude ratio at the static level and the ratio of depth burial [Mueller and Murphy, 1971; Kim et al., 2009]:

$$\left(\frac{W_1}{W_2}\right) = \left[\frac{A'_1}{A'_2} \left(\frac{h_1}{h_2}\right)^{0.33}\right]^{0.87} = \left[A_r \left(\frac{h_1}{h_2}\right)^{0.33}\right]^{0.87} \quad (1)$$

where W is yield, A' is the displacement at low frequency asymptote level without a correction on the attenuation, h is source depth and Ar is the ratio of A' of two explosions. The Ar is equivalent to a source scaling at low frequency. A variation of yield ratio can be calculated with a varying ratio of source depth with amplitude ratios at static level.

### Corner frequency ratio

Corner frequencies are expressed mathematically as

$$f_c = \frac{1}{2\pi} \sqrt{\frac{\Omega}{A}} \quad (2)$$

Where A is the low frequency asymptote level estimated from displacement spectra and  $\Omega$  is the high frequency asymptote level estimated from acceleration spectra [Glassmoyer and Borchardt, 1990]. Both displacement and acceleration spectra need to be corrected for the attenuation before the asymptote estimation. If we take corner frequency ratio between two explosions, the ratio fr, however, can be given in terms of ratio of each asymptote without the correction on attenuation because the asymptote ratio cancels out attenuation term;

$$f_r = \frac{f_{c1}}{f_{c2}} = \sqrt{\frac{\Omega_r}{A_r}} \quad (3)$$

$\Omega_r$  is identical to a source scaling at high frequency between two events nearly co-located while Ar is that of low frequency.

Corner frequencies can be also represented with P wave velocity of the source medium Vs(P) and the elastic radius Re;  $f_c = (Vs(P))/(2\pi Re)$  [Denny and Johnson, 1991; Fisk, 2006]. The corner frequency ratio from this representation gives another relationship in terms of the ratio between different elastic radii. Based on the empirical studies, the elastic radius Re has a proportional relationship to  $W^{(1/3)}[(\rho_s g h)]^{-0.417}$  for granite, tuff/rhyolite and shale where W is yield,  $\rho_s$  is density of medium at source location, g is gravitational acceleration and h is source depth [Fisk, 2007]. If this proportional relationship is applied to the DPRK's test site, the representation of corner frequency ratio can be described as [Mueller and Murphy, 1971];

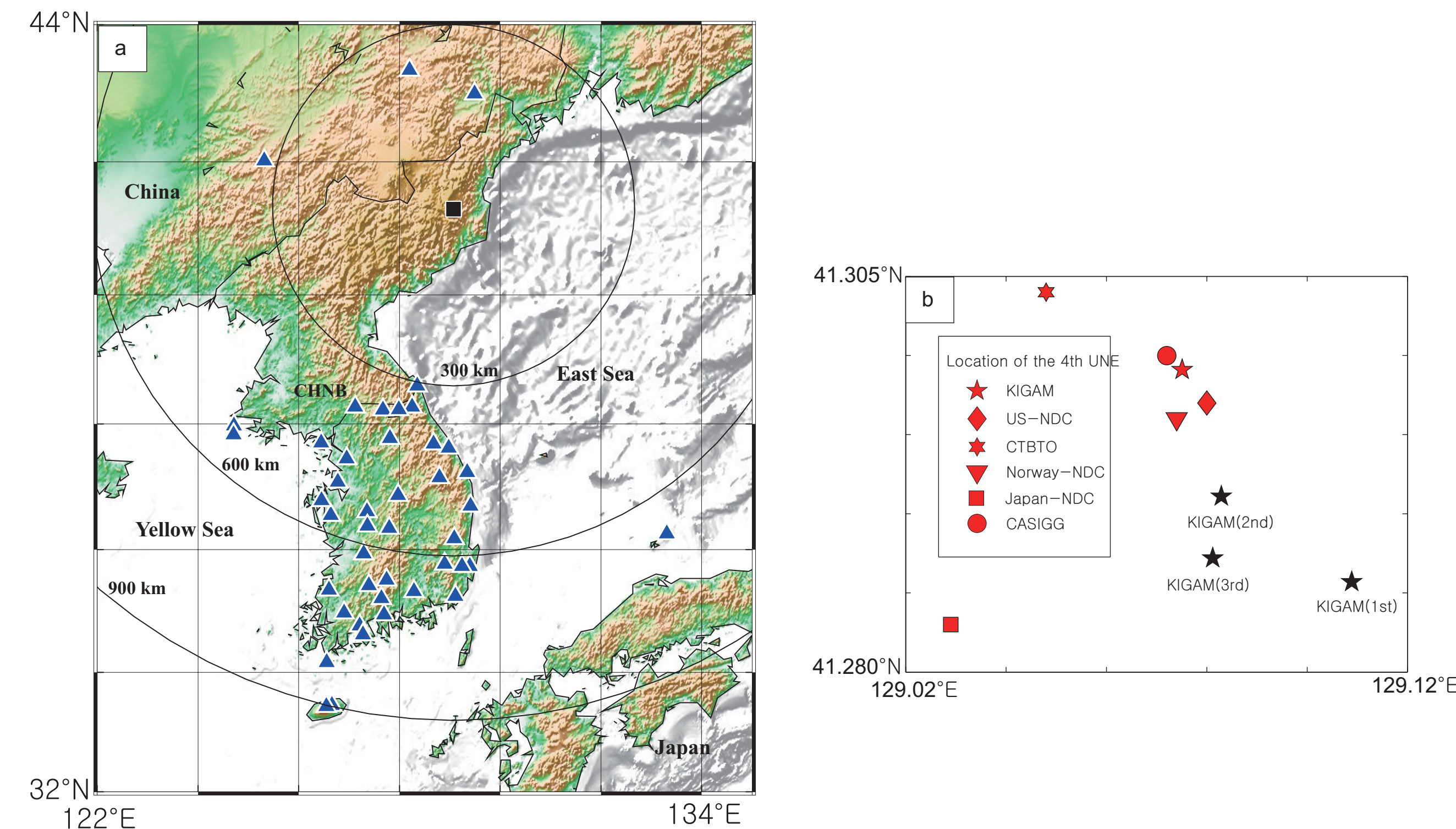
$$f_r = \frac{f_{c1}}{f_{c2}} = \frac{R_{e2}}{R_{e1}} = \frac{W_2^{1/3}(\rho_s g h_2)^{-0.417}}{W_1^{1/3}(\rho_s g h_1)^{-0.417}} = \left(\frac{W_2}{W_1}\right)^{1/3} \left(\frac{h_2}{h_1}\right)^{-0.417} \quad (4)$$

The corner frequency ratio, fr, can be represented by a function of yield ratio and source depth ratio. The power -0.417 was adopted from empirical studies from past nuclear tests in granitic medium [Mueller and Murphy, 1971; Fisk 2007]. The rock type in the test site is known as Bulguksa granite generated by igneous activities during the Bulguksa Disturbance occurred from Cretaceous period to early Cenozoic Era [Lee, 1988].

From (3) and (4), the yield ratio is represented by the corner frequency ratio and source depth ratio.

$$\left(\frac{W_1}{W_2}\right) = \left(\frac{f_{c1}}{f_{c2}}\right)^{-3} \left(\frac{h_1}{h_2}\right)^{1.251} = \left[\left(\frac{\Omega_r}{A_r}\right)^{0.5}\right]^{-3} \left(\frac{h_1}{h_2}\right)^{1.251} \quad (5)$$

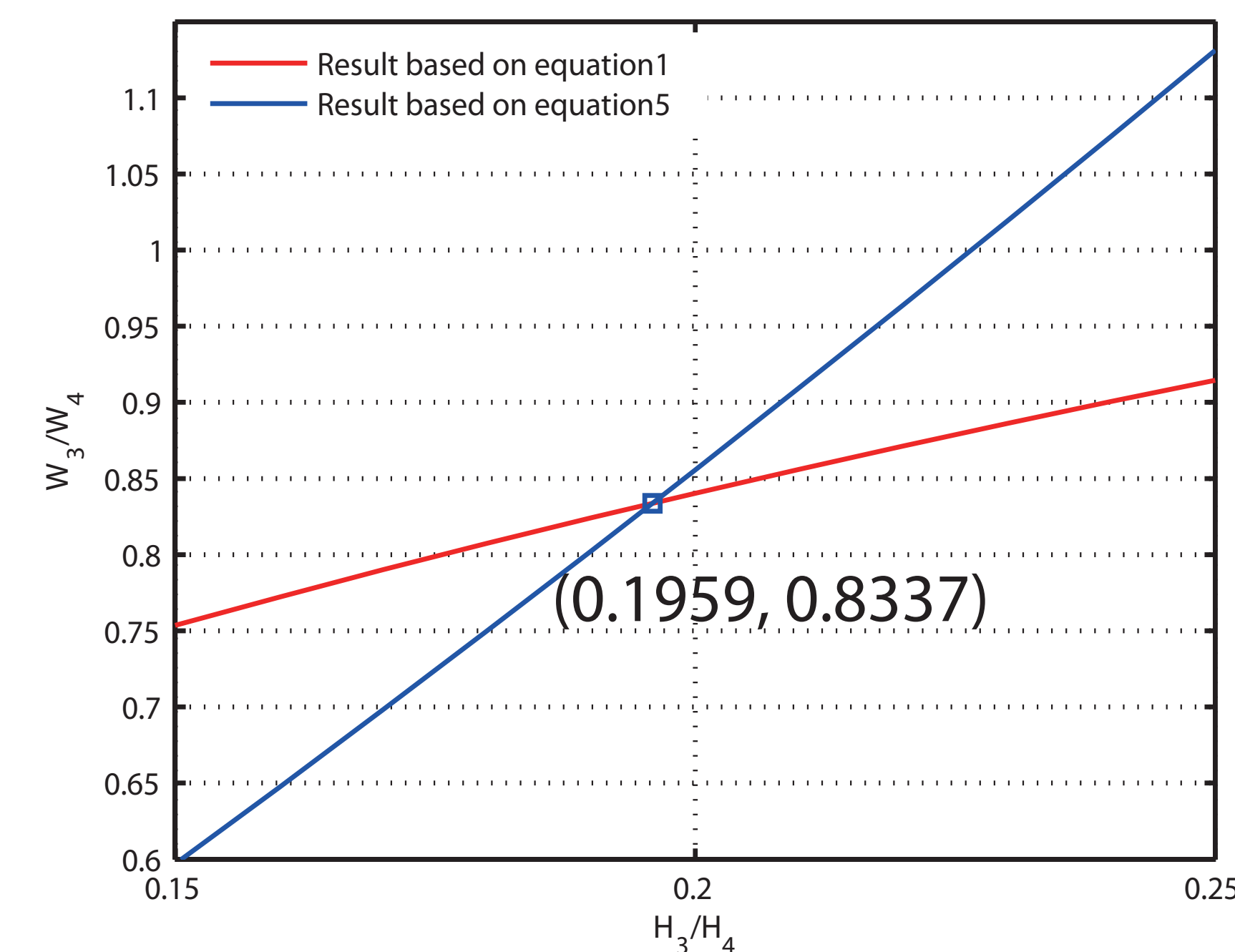
## IV. Stations and Event Location



Geographical map. (a) Location of the DPRK's test site (black rectangle), seismic stations of Korea, China and one station in Japan (blue triangle). (b) Different organizations' location results for the DPRK's fourth UNE are compared to each other (red polygons) with KIGAM's locations on the other three UNEs (black star).

The fourth event was located in the DPRK's UNE test site in the P'unggyeri where the other three UNEs were also detonated at a close distance range within about 2 km. Figure illustrates the fourth UNE's location estimated by different organizations, which shows four locations are closely clustered.

## VI. Depth Ratio and Yield Ratio



The two lines from the solution of equation 1 and 5 cross each other. The most probable solution of depth ratio and yield ratio between the third and fourth is the crossed point of the two curves.

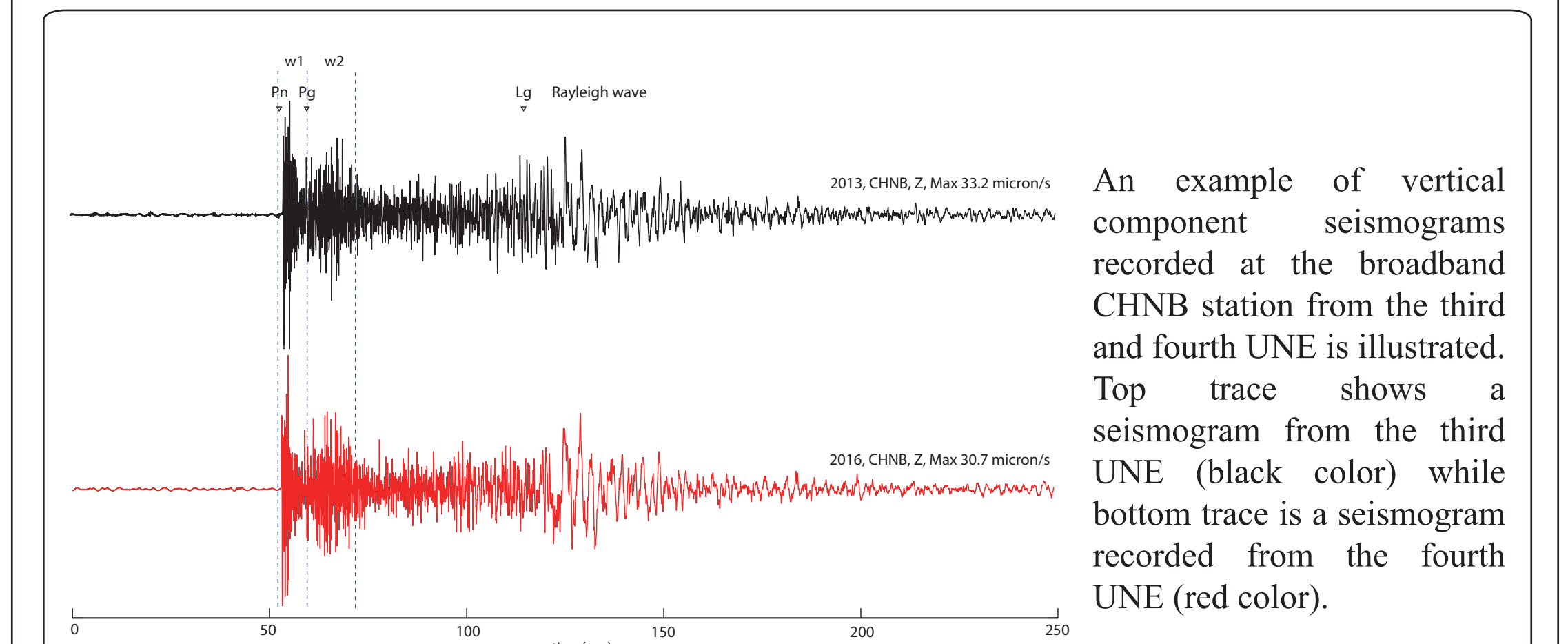
It indicates the depth of the fourth UNE is 5 times deeper than that of the third one while the yield of the fourth UNE is 1.2 times larger than that of the third UNE.

## VII. Conclusion

DPRK's fourth UNE was performed at the P'unggyeri test site in which closely located other four events had been carried out. The geological effect in source medium and the attenuation effect along the propagation path are eliminated by calculating spectral ratio between the DPRK's third (mb 4.9, KIGAM) and fourth UNE (mb 4.8, KIGAM). This process is based on an assumption of fully coupled explosions for the closely located events in a granitic medium.

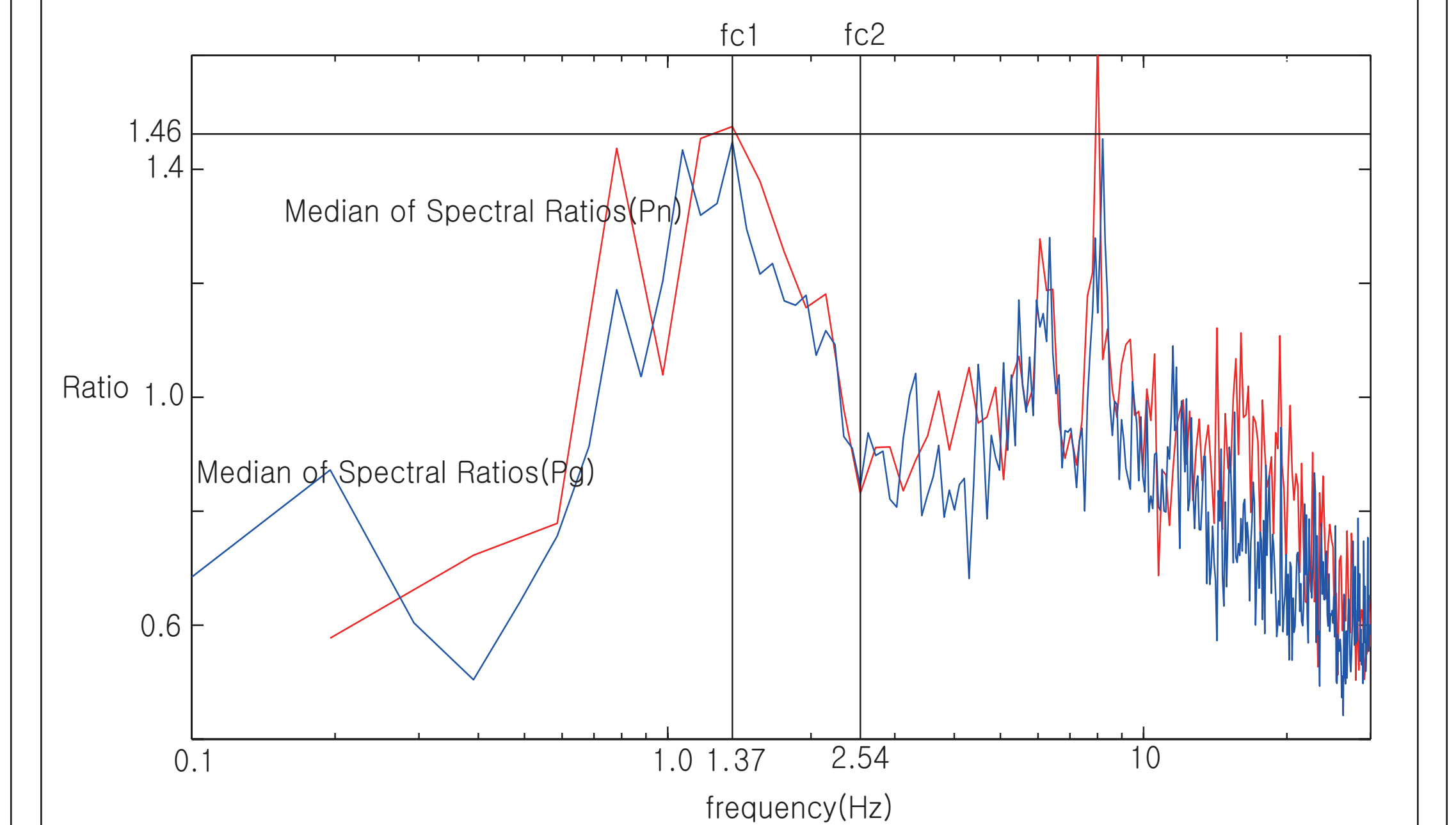
The third UNE's spectrum is divided by that of the fourth UNE. Spectral Ratios for the Pn and Pg phases are calculated for fifty 3-component data sets. The median of spectral ratio is estimated. The ratio asymptote at low frequency is estimated as 1.46 which is used for calculating a numerical solution of the equation 1. The first corner frequency, fc1, is estimated as 1.37 Hz while the second one is, fc2, is estimated as 2.54 Hz. The corner frequency ratio fc1/fc2 is used for calculating a numerical solution of the equation 5.

## V. Amplitude Ratio and Corner Frequency Ratio Calculated from Data



An example of vertical component seismograms recorded at the broadband CHNB station from the third and fourth UNE is illustrated. Top trace shows a seismogram from the third UNE (black color) while bottom trace is a seismogram recorded from the fourth UNE (red color).

A 5.12 second-signal window is selected for the Pn phase with high sampling rate of 100 sps (w1) while a 10.24 second-signal window was selected for the Pg phase (w2). The same length of noise window before the first arrival is selected for the estimates of background noise level. 10% cosine taper is applied to the segmented data.



The third UNE's spectrum is divided by that of the fourth UNE. Spectral ratios for the Pn and Pg phases are calculated with fifty 3-component data sets. The median of spectral ratios is estimated for Pn and Pg as illustrated above.

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The first corner frequency, fc1, is estimated as 1.37 Hz while the second one is, fc2, is estimated as 2.54 Hz. The corner frequency ratio fc1/fc2 is used for calculating a numerical solution of the equation 5.

A relative depth and yield ratio is attained by solving two equations utilizing amplitude ratio at low frequency and corner frequency ratio. The intersected point of the two equations indicates the depth of the fourth UNE's source is 5 times deeper than that of the third one while the yield of the fourth UNE is 1.2 times bigger than that of the third UNE.