Incorporating realistic terrain boundary conditions into numerical infrasound propagation modelling

Codor Khodr[1], David Green[2] and Mahdi Azarpeyvand[1]

I . Introduction

Infrasound is a sound wave with a low frequency (typically $f \leq 20$ Hz). Infrasound sources include global events such as volcanic eruptions, earthquakes and nuclear explosions.

Infrasound is one of the waveform technologies used to monitor compliance of the Comprehensive Nuclear-Test-Ban Treaty (CTBT). Therefore, the accurate prediction of infrasound propagation over realistic terrain is of significant interest.

Most numerical models for acoustic propagation in the atmosphere (FFP, PE, Normal Modes, Broadband Modelling, Ray Tracing) neglect the influence of boundary topography whereas medium inhomogeneity has been extensively studied. A combination of both effects is expected to improve modelling capability.

II. PE method for Infrasound

The Parabolic Wave Equation (PE) method is a numerical method for the computation of wave propagation in a fluid or elastic medium. Through the 20th century, it has been applied to electromagnetics and ocean acoustics before atmospheric acoustics (Lee et al., 2000). It is obtained from the paraxial (small angle) approximation of the frequency-domain Helmholtz equation:

$$\Delta p_c(r,\varphi,z) + k_0^2 n^2(r,\varphi,z) p_c(r,\varphi,z) = 0$$

It is reduced to a one-way wave equation at far-field ($k_0 r \gg 1$):

$$q_c = p_c\sqrt{r} \qquad \Psi(r,z) = q_c(r,z)e^{-ik_0 r} \qquad \left(\frac{\partial}{\partial r} + ik_0(1-\sqrt{1+\sigma})\right)\Psi(r,z) = 0$$

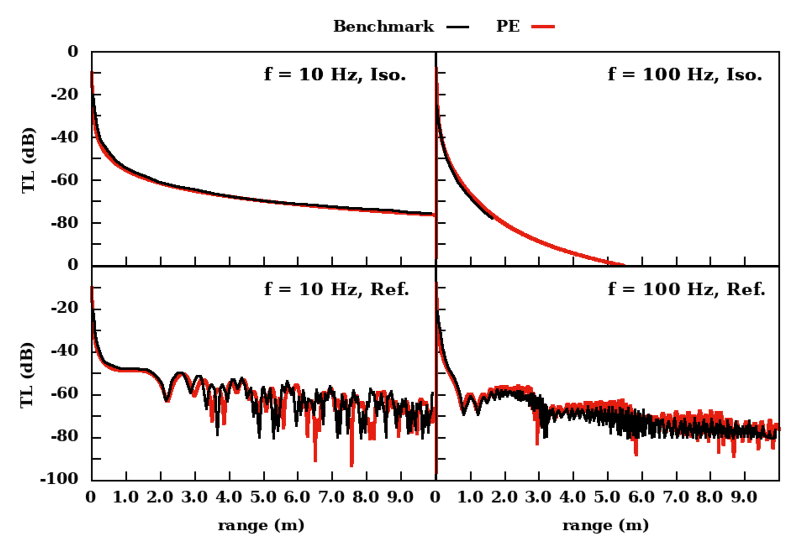
The resulting pseudo-differential operator is approximated by Taylor expansion:

$$\sigma = \frac{1}{k_0^2}\frac{\partial^2}{\partial z^2} + \delta n^2(z) \qquad \left(\frac{\partial}{\partial r} - ik_0\frac{\sigma}{2}\right)\Psi(r,z) = 0$$

The propagation domain is then discretized using finite-differences, leading to a forward-marching procedure:

$$\left(\mathbf{I} - \frac{\Delta r}{2}(a\mathbf{T}+\mathbf{D})\right)\vec{\psi}_{m+1} = \left(\mathbf{I} + \frac{\Delta r}{2}(a\mathbf{T}+\mathbf{D})\right)\vec{\psi}_m$$

A validation procedure against Attenborough et al. (1995) has been undertaken for different frequencies over a flat impedance plane. The quantity of interest for far-field sound propagation is the transmission loss, obtained from the relation below.



$$TL(r,z) = 10\log\left(\frac{p_c(r,z)^2}{p_{ref}^2}\right)$$

Fig. 1 – Transmission loss at the ground, for $f = 10$ Hz (left column) and $f = 100$ Hz (right column), in an isothermal (top line) and downward refracting atmosphere (bottom line). The PE code implemented shows very good agreement with the benchmarks.

III. Influence of Topography on sound propagation

Topographic effects on infrasound propagation include diffraction, back-scattering, reflection and caustics (shadow zones). For shallow slopes, the ground impedance can be assumed to remain constant.

A shift map method is used in order to integrate the terrain profile inside the 2D PE method, the PE is simplified for the appropriate wave modulator Ψ (Donohue and Kuttler, 2000)

$$\begin{cases}\xi = r \\ \eta = z - H(r)\end{cases} \Longrightarrow \begin{cases}\partial/\partial r = \partial/\partial\xi - H'(\xi)\partial/\partial\eta \\ \partial/\partial z = \partial/\partial\eta\end{cases}$$

A new PE is obtained, which is commonly referred to as the BTPE (Beilis-Tappert PE):

$$\left(\frac{\partial}{\partial\xi} + ik_0\left(G(\xi) - \sqrt{1+\sigma}\right)\right)\Psi(\xi,\eta) = 0$$

Finally, the numerical procedure changes to:

$$\left(\mathbf{I} - \frac{\Delta r}{2}(a\mathbf{T}+\mathbf{D}-\mathbf{B}_m)\right)\vec{\psi}_{m+1} = \left(\mathbf{I} + \frac{\Delta r}{2}(a\mathbf{T}+\mathbf{D}-\mathbf{B}_m)\right)\vec{\psi}_m$$

The PE model developed can accommodate slopes up to $\theta = 30°$, a reasonable limit for real terrains. Wide-Angle PE models (WAPE) can increase this limit.

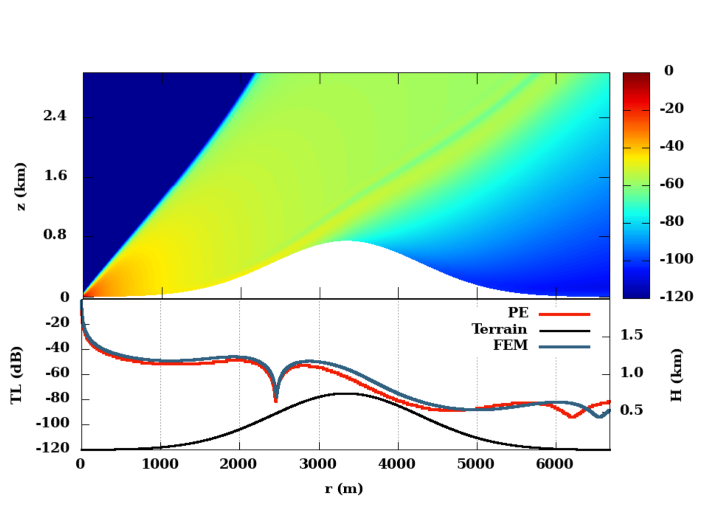


Fig. 2 – Wave propagation over a Gaussian hill: transmission loss field (dB) in the atmosphere (top) and along the ground surface (bottom) for a frequency $f = 10$ Hz and an isothermal atmosphere. The source height is 5.0 m. Comparison of the PE against a finite-element model (COMSOL). The discrepancies are due to the definition of the source: the PE uses a Gaussian bell curve to approximate the Dirac impulse $\delta = \delta(z - z_s)\delta(r)$ in the near-field. (Salomon, 2001)

IV. Ascension Island Example: Data

Ascension Island hosts the International Monitoring System (IMS) Infrasound array IS50 (cf. CTBTO Spectrum 9), comprising eight microbarographs with an aperture of 2700 m, four of which are low-frequency elements (L1-4).

The presence of a wind farm (W) at 7.961S, 14.387W suggests that the peaks observed in the amplitude spectrum (Fig. 3c) are caused by the variation of air pressure generated by the blade rotation past the turbine tower. A simple study shows that this observation is consistent with the turbine technical specifications (Micon M700-225). As a consequence, the following frequencies are retained for the PE simulation:

Harmonic	Fundamental	1st Overtone	2nd Overtone	3rd Overtone
Frequency (Hz)	1.90	3.79	5.69	7.59

The Microbarographs L1, L2, L3 and L4 are located at a distance of $l_1 = 2890$ m, $l_2 = 3240$ m, $l_3 = 4580$ m and $l_4 = 1830$ m from the wind farm (W) respectively.

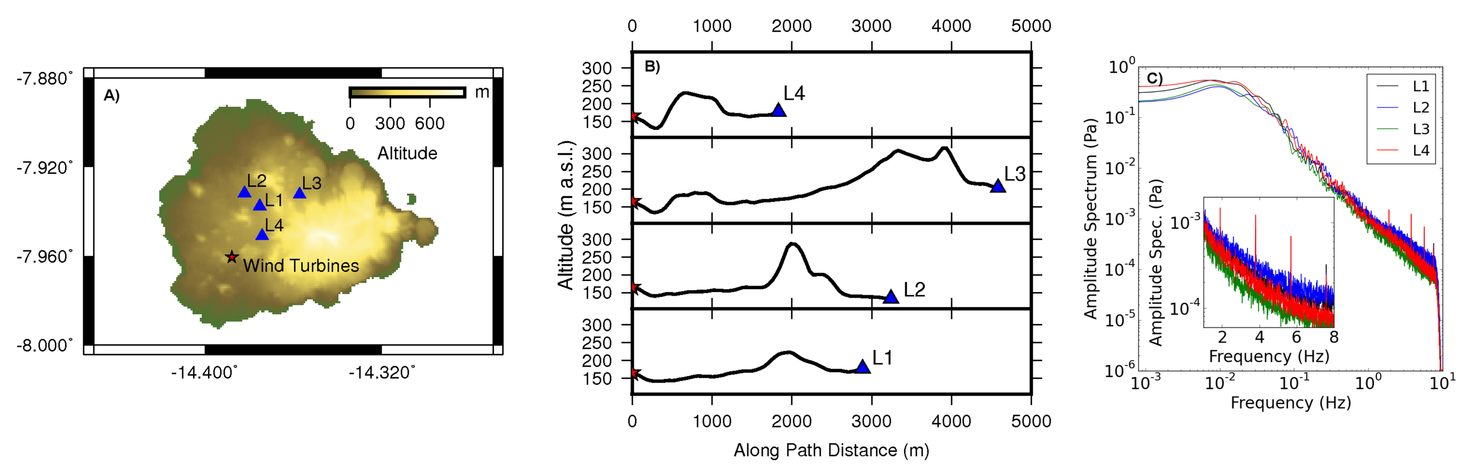


Fig. 3 – a) Map of Ascension Island with location of the Wind Turbines and the IS50 low-frequency microbarographs. b) Topographic profiles of the paths (W-L1) to (W-L4) between the wind turbines and the microbarographs. c) Amplitude spectra for the four microbarographs recorded in October 2010.

V. Ascension Island Example: Modelling

The PE method is used to model the wave propagation, the pressure at the receiver across the four paths between (W-L1), (W-L2), (W-L3) and (W-L4) is calculated from the PE simulations and compared against the International Data Centre (IDC) recordings.

The transmission loss at the ground level shows that higher frequencies are more subject to terrain-induced effects. In order to quantify the latter, a natural approach involves the comparison of the PE and data against the one obtained for a flat terrain. A straightforward analysis using the analytical solution of the Helmholtz equation in free field give the following amplitude ratios in the flat case, for all frequencies:

Case	L_2/L_1	L_3/L_1	L_4/L_1
Ratio	0.89	0.63	1.58

The amplitude ratios are calculated with respect to a reference pressure, which is chosen to be the pressure at the L1 microbarograph, for each frequency :

$$\frac{P}{P_1} = 10^{\frac{\mathrm{TL}-\mathrm{TL}_1}{20}}$$

The pressure at the microbarographs is calculated from the transmission loss. The results show good agreement with data, except for the first and third overtone over the path (W-L4).

The present simulations assume an isothermal atmosphere i.e. with a constant sound velocity ($c_0 = 343.3\ \mathrm{m.s^{-1}}$), as well as a rigid ground boundary (infinite impedance). More realistic atmosphere condition can be implemented to reach a better understanding of terrain effects (Fig. 6).

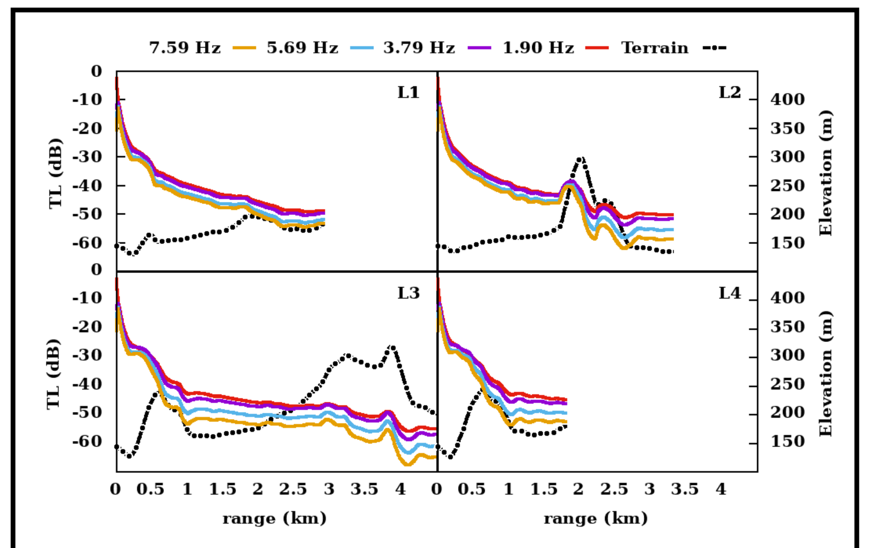


Fig. 4 – Wave propagation between the Wind Farm and the four microbarographs located at L1, L2, L3 and L4, for the fundamental frequency $f_0 = 1.90$ Hz and the three overtones $f_1 = 3.89$ Hz, $f_2 = 5.79$ Hz and $f_3 = 7.59$ Hz. The atmosphere is assumed to be isothermal ($c(z) = c_0 = 343$ m/s)

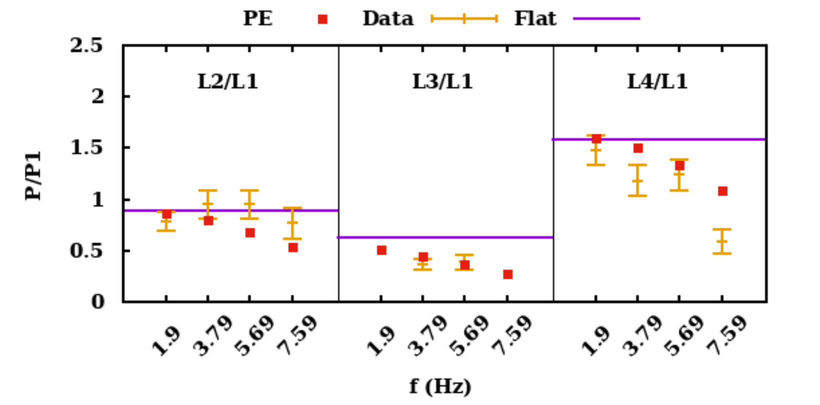


Fig. 5 – Amplitude ratios at the receivers L2, L3 and L4 relatively to L1, for f_0, f_1, f_2 and f_3. Comparison between IDC Data recorded at IS50 during October 2010, the PE simulations and the flat terrain analysis

VI. Future work

The next step is to extend the present method to 3D wave propagation, so the lateral wave propagation (due to terrain coupling) can be taken into account.

Another concern is the physical accuracy of the PE :

- Angle capability can be increased by using higher order Padé expansion
- Backscattering can be included thanks to the two-way PE
- Mode coupling can be implemented in the so-called "Coupled-Mode PE"

Terrain effects will be further assessed by doing a comparison of the PE code against experiments in the University of Bristol's anechoic chamber. A set of sample terrains will be used for scaled linear acoustic simulations. The measurements will serve as a benchmark for future wave propagation models

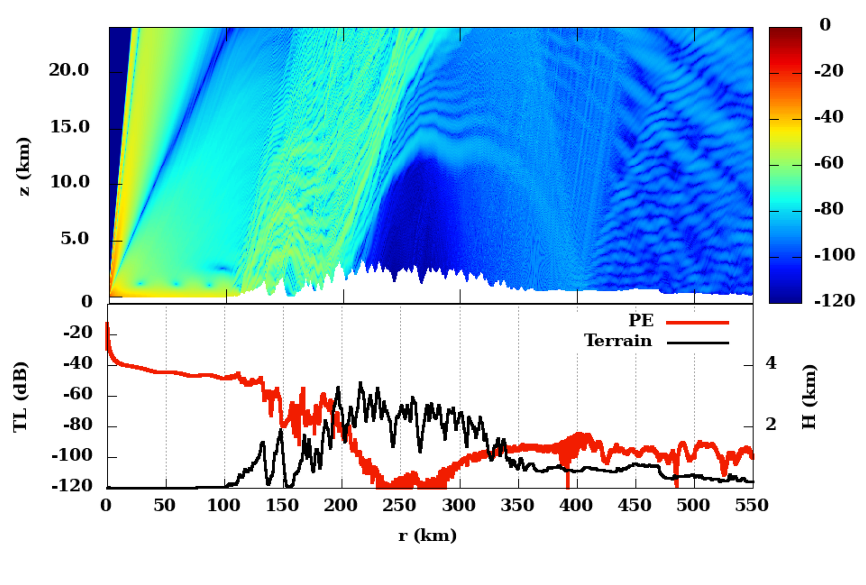


Fig. 6 – Wave propagation over a 2D topographic path in the Alps region: transmission loss field (dB) in the atmosphere (top) and along the ground surface (bottom) for a frequency $f = 1$ Hz and a downward refracting atmosphere. The source height is 250.0 m. The combination of wind and topography creates a wave "bounce" from about $x = 200$ km to about $x = 400$ km. The roughness in the topographic data suggests that it may need to be filtered according to the wavelength.

D. Lee et al. Parabolic equation development in the Twentieth Century. *Journal of Computational Acoustics*. **8** (4) 527-637, 2000.
K. Attenborough et al. Benchmark cases for outdoor sound propagation models. *JASA*. **97** (1) 173-191, 1995.
D.J. Donohue and J.R. Kuttler. Propagation Modelling over terrain using the Parabolic Wave Equation. *IEEE Transactions on Antennas and Propagation*. 48 (2) 260-277, 2000.
E.M. Salomon. Atmospheric Acoustics. *Kluwer Academic Publishers*. 2001.

[1] University of Bristol, Bristol, UK
[2] AWE Blacknest, Brimpton, UK