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A Compressive Sensing and Sparsity Based Method for Time-Frequency Distributions Optimisation

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Abstract

Nonstationary signals are optimally represented in the joint time-frequency domain using time-frequency distributions (TFDs). The unwanted artefacts, which are by-products of TFDs quadratic nature, make TFD interpretation a challenging task.
Recently proposed methods address the problem of artefact removal by employing compressive sensing (CS) techniques, with unavoidable resolution loss being reduced by using reconstruction algorithms based on sparsity constraints.
In this work, we study the effects of the CS area selection on the resulting sparse TFD performance. We also propose a method for an automatic data-driven CS area selection.
The method performance is tested on synthetic and real-life signals, including examples of geophysical signals models.

Quadratic Time-Frequency Distributions

- Time-frequency distributions (TFDs): observe signal energy distribution as a function of both time and frequency simultaneously.
- Consider a multi-component linear frequency modulated (LFM) signal $z(t)$, with N_c components:

$$z(t)=\sum_{i=1}^{N_c} A_i(t)e^{j\varphi_i(t)},$$

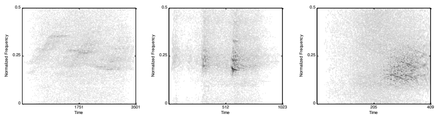
  $A_i(t)$, $\varphi_i(t)$ are the amplitude and phase of the i-th component.
- Ideal signal time-frequency distribution is a set of Dirac functions:

$$\hat{\rho}(t,f)=\sum_{i=1}^{N_c} A_i^2(t)\delta(f_i-f_{i0}(t)),$$

  $f_{i0}(t) = (1/2\pi)d\varphi_i(t)/dt$ is the instantaneous frequency (IF) of the i-th component.
- In real-life applications, signal analytical form is not obtainable; numerical methods for TFD calculation.
- Wigner-Ville distribution (WVD): perfect localization for a mono-component LFM signal:

$$W_z(t,f)=\int_{-\infty}^{\infty} R_z(t,\tau)e^{-j2\pi f\tau}d\tau$$

  $R_z(t,\tau) = z(t+\tau/2)z^*(t-\tau/2)$ is the signal local autocorrelation function (LAF).
- WVD drawback: when the signal has $N_c>1$, cross-terms appear between each pair of components.
- Cross-terms are highly oscillatory and can be filtered out in the ambiguity function (AF), which is the Fourier transformation of the TFD.
- Auto-terms also get partially filtered out, hence reducing the concentration of the auto-terms in the time-frequency plane.



Whale signal WVD — Explosion signal model WVD — Earthquake signal model WVD

- Since there is no single best performing kernel for all signals, the need to adaptively construct kernel has arisen.
- Radially Gaussian Kernel (RGK) solves the following optimization problem:

$$g_{opt}(\upsilon,\tau)=\arg\min\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|A_z(\upsilon,\tau)g(\upsilon,\tau)\right|^2 d\tau d\upsilon.$$

TFD as a Sparsity Inducing Domain

- Compressive sensing (CS) provides an alternative way to deal with the cross-terms, without the resolution loss.
- Ideal TFDs are inherently sparse, since they are composed of the components IFs.
- The sensing matrix, $\phi(\nu,\tau)$, is designed in a way which discards highly oscillatory cross-terms in the AF:

$$A_z'(\nu,\tau)=\phi(\nu,\tau)A_z(\nu,\tau),$$

  $A_z'(\nu,\tau)$ is the CS-AF, and $A_z(\nu,\tau)$ is the AF.
- In the standard CS notation, the Fourier transformation is denoted as multiplication with matrix ψ:

$$A_z'(\nu,\tau)=\psi\cdot\vartheta_z(t,f),$$

  $\vartheta_z(t,f)$ is the reconstructed TFD.
- This leads to under-determined system: goal of the reconstruction algorithm is to find an optimal solution to:

$$\vartheta_z(t,f)=\psi^H A_z'(\nu,\tau).$$

- The problem is a well known unconstrained optimization problem and can be solved by minimizing the ℓ_1 norm :

$$\vartheta_z^{l_1}(t,f)=\arg\min\left\|\vartheta_z(t,f)\right\|_1,\text{s. t. }\left\|\vartheta_z(t,f)-\psi^H A_z(\upsilon,\tau)\right\|_2^2\le\varepsilon.$$

$$\left\|\vartheta_z(t,f)\right\|_1=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|\vartheta_z(t,f)\right|dtdf.$$

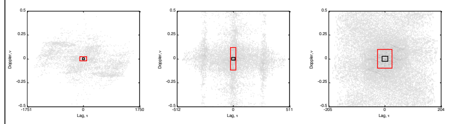
- The ℓ_1 norm minimization has a unique closed form:

$$\vartheta_z^{l_1}(t,f)=\text{soft}(\vartheta_z(t,f),\lambda),$$

  λ is the regularization parameter.

CS-AF Area Selection

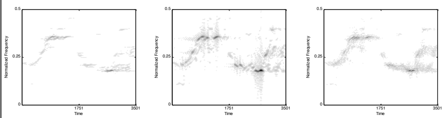
- Selection of the CS-AF area is crucial for the TFD localization and cross-terms suppression.
- The CS–AF area is usually chosen experimentally as a rectangle containing $N'_\tau \times N'_\upsilon \approx N_t$ samples, centered at the AF domain origin.
- The goal is not to exactly reconstruct the starting TFD; the goal is to obtain a new WVD-like TFD, with highly suppressed cross-terms.
- The general goal of the proposed adaptive CS-AF area selection method is to capture as large as possible area around the AF origin, without including any cross-terms.
- This is achieved by searching the zero-doppler and the zero-lag AF slices for points where the cross-terms intersect with the respective AF axis.



Whale signal CS-AF — Explosion signal model CS-AF — Earthquake signal model CS-AF

Examples

Example 1: Whale signal



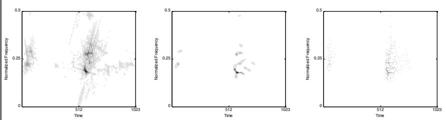
RGK-TFD — Sparse TFD reconstructed with NESTA and fixed CS-AF — Sparse TFD reconstructed with NESTA and auto CS-AF

- TFD performance is measured using the concentration measure, which has lower values for better concentrated TFDs:

$$M_z^S=\frac{1}{N_t N_f}\left[\sum_i\left(\vartheta_{zN}(t,f)\right)^{1/2}\right]^2.$$

		NESTA (ε=10%)	NESTA (ε=1%)	NESTA (ε=0.1%)	RGK (α=3)
M_z^S	FIX	0.5406	0.5406	0.5406	1.2248
	AUTO	0.4877	0.4877	0.4877	
t [s]	FIX	45.7979	46.7524	46.0662	6.6070
	AUTO	45.9895	45.8414	45.8758	

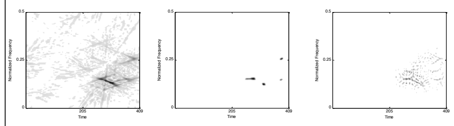
Example 2: Explosion signal model



RGK-TFD — Sparse TFD reconstructed with NESTA and fixed CS-AF — Sparse TFD reconstructed with NESTA and auto CS-AF

		NESTA (ε=10%)	NESTA (ε=1%)	NESTA (ε=0.1%)	RGK (α=3)
M_z^S	FIX	0.4011	0.3950	0.3895	2.8401
	AUTO	0.3720	0.3644	0.3524	
t [s]	FIX	2.4230	6.8978	18.3168	0.7685
	AUTO	2.6934	8.1296	22.8794	

Example 3: Earthquake signal model



RGK-TFD — Sparse TFD reconstructed with GPSR and fixed CS-AF — Sparse TFD reconstructed with GPSR and auto CS-AF

		GPSR (ε=10%)	GPSR (ε=1%)	GPSR (ε=0.1%)	RGK (α=3)
M_z^S	FIX	0.0050	0.0050	0.0033	1.8974
	AUTO	0.0220	0.0220	0.0148	
t [s]	FIX	0.2142	0.1812	0.3490	0.1880
	AUTO	0.1809	0.1803	0.3516	

Conclusion

- The presented method adaptively detects the CS-AF area which guaranties the optimal amount of input data for sparse reconstruction algorithms.
- The method results in highly concentrated TFDs, with fast sparse reconstruction algorithm convergence.
- The effectiveness of the proposed method has been illustrated on synthetic and real-life signals examples.

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