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Abstract. We consider a hypercomplex and high-order (tensorial) representation of higher than 4 dimensional algebras, or some specific grouping of them, so tensor multichannel seismic data as they are recorded by the International Monitoring System of representation of seismic wavefield looks natural in this case. Data processing then would be conducted not on separate waveform projections but on relatively full multidimensional multidimensional model. In particular, we discuss an approach to construct object and tensor operations on the data from the 3C arrays would utilize joint volumetric multidimensional master event templates for further cross-correlation-based detection and (sensor) and spatial (array) information. Further dimensionality reduction of tensor data location with the data recorded at multichannel seismic installations, such as IMS 3-C produces lower order principal components, a basis for the multidimensional waveform seismic stations and arrays. Hypercomplex number systems are the natural cases of templates. Highly effective master events built with the hypercomplex and multilinear SVD representing a 3-C digital seismogram samples requiring however special attention to the underlying axiomatics. Dealing with the composite observations (3C arrays) may demand

Conclusions. This presentation is an introduction to the multidimensional mathematical approach to representation of multichannel seismic network data as hypercomplex and tensor constructions on the example of primary and auxiliary stations of International Monitoring System with further goal to understand the profit it can bring in tasks of relative location for CTBT monitoring. We studied the benefits and the caveats of using hypercomplex and multilinear Singular Value Decomposition methods applied to such construction, and discovered effective measures of correlating hypercomplex data structures. For the routine applications we apply so far simplified approach based on creation of master events with one-dimensional vectors extracted from tensorial structures (eigenimages or reduced structures). Even if such restricted form it works more efficiently than conventional one-dimensional cross-correlation approach. There are still some problems using these approaches, connected with dimensionality transformations for hypercomplex constructions and temporal synchronizing for both hypercomplex and tensorial structures. Application of commutative algebras approaches should also be studied along with classical quaternions. The benefits of multilinear SVD comparing with lower cost dimensionality reduction methods (interpolation, discrete Fourier transform, etc.) should also be studied.

Introduction. In this poster, we discuss the approach to construct master event templates for further cross-correlation-based location with the data recorded at multichannel seismic installations, such as 3-C seismic arrays of the International Monitoring System (IMS) of the CTBTO. This approach can be extended to other so called "vector sensor" applications such as time lapse seismic, hydraulic fracturing monitoring, or reservoir characterization.

The objectives for this study are:

- Need to pursue the most effective dimensionality reduction methods for master event template design in global seismic monitoring
- Hypercomplex data representation is the natural way of presenting 3D-polarized data
- Successful application of tensorial and hypercomplex approaches in geophysical (exploration) multidimensional data processing
- Lack of such experience in observational seismology

A multicomponent seismic sample can be represented, in general case, as a hypercomplex number, a quaternion as a special case. A pure quaternion of order 4 and other hypercomplex systems are the natural cases of representing a 3-component digital seismogram sample requiring however special attention to the non-commutativity or non-transitivity of the corresponding algebra. Dealing with more complex observations within the approach of assigning the number's dimension to the observed physical dimension (more precisely, to the measurement linked with the dimension) demands higher than 4 dimensional algebras, or some specific grouping of them. Dealing with the tensor representation of seismic wavefields can simplify in certain sense the multidimensional approach to data processing, in particular, to the data set which was the same as used for the single component processing. It is shown in our poster T3.7-P6 that single event multichannel data corresponding to a seismic event can be rearranged as a 3 or 4 (for 3C arrays) mode tensor, where first mode is time, or sample number, the second mode is station, or sensor number, and the third mode is the direction of ground motion (Z, N and E). Then we could apply tensor operations to the data recorded by such arrays gaining certain benefits from utilizing joint volumetric (sensor) and spatial (array) information. Further dimensionality reduction of tensor data produces low order principal components, a basis for the multidimensional waveform templates (see also our poster T3.7-P6).

Basics of quaternionic approach

To our knowledge, first evidence of using hypercomplex algebraic systems in geophysics can be referred to the set of publications in Doklady Akademii Nauk, USSR back in 1983. An associative commutative hypercomplex system of order 4 was introduced by Shpilker (1983-1988), and then expanded to the systems of theoretically unlimited order (ascending to the works of Segre (1892) and Ketchum (1928) on commutative infinite dimensional algebra) in order to solve incorrect (or incorrectly formulated) problems in Hadamard sense. With this, in contrast to classical quaternion extensions, octonions and sedenions, Shpilker's algebra was associative though not transitive and can also be considered as a superalgebra. A seismological application was considered by Rozhkov (1986) in a study of hypercomplex wave field restoration with data recovered by homomorphic analysis.

Historically, W. Hamilton (1843) made the first attempts to construct an algebra associated with the three-dimensional Laplace equation. He constructed an algebra of noncommutative quaternions over the field of real numbers R and made a base for developing the hypercomplex analysis.

In a modern world quaternions are getting more and more popular: computer vision and colour image processing, missile flight control, robotics (with quaternion neural networks), exploration geophysics (vector sensor arrays), quantum mechanics, special and general relativity.

Quaternions are an extension of complex numbers from the 2D plane to the higher spaces and form one of the four existing division algebra (real R, complex C, quaternions H and octonions). A quaternion q is an element of the 4D normed algebra over the real number with basis {1; i; j; k}. So, q has a real part and three imaginary parts:

$$i^2 = j^2 = k^2 = ijk = -1 \quad \mathbf{q} = \langle s, \mathbf{v} \rangle \quad \text{For unit vector quaternion:} \quad \text{The dot product of two quaternions works in the same way as the dot product of two vectors:}$$

$$i = jk = -kj \quad s = q_0 \quad \mathbf{q} = [q_0 \quad q_1 \quad q_2 \quad q_3]$$

$$j = ki = -ik \quad \mathbf{v} = [q_1 \quad q_2 \quad q_3] \quad |\mathbf{q}| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1 \quad \mathbf{p} \cdot \mathbf{q} = p_0q_0 + p_1q_1 + p_2q_2 + p_3q_3 = |\mathbf{p}||\mathbf{q}|\cos\varphi$$

$$k = ij = -ji \quad \mathbf{q} = q_0 + iq_1 + jq_2 + kq_3$$

For Hilbert space, a quaternionic discretized signal of N samples is a vector whose elements are quaternions:

$$\mathbf{x} = [x_1 \ x_2 \ \dots \ x_N]^T \in \mathbb{H}^N \quad \text{and} \quad x_i \in \mathbb{H} \quad \text{Over this vector space, a scalar product can be defined as}$$

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^c \mathbf{y} = \sum_{\alpha=1}^N \bar{x}_\alpha y_\alpha \quad \text{where } \bar{\cdot} \text{ denotes the quaternion transposition-conjugate operator. A metric can also be defined in order to measure the distance between two quaternion vectors } \mathbf{x} \text{ and } \mathbf{y}: \langle \mathbf{x}, \mathbf{y} \rangle = \|\mathbf{x} - \mathbf{y}\| = [(\mathbf{x} - \mathbf{y})^c (\mathbf{x} - \mathbf{y})]^{1/2}, \text{ and quaternion SVD can be defined as}$$

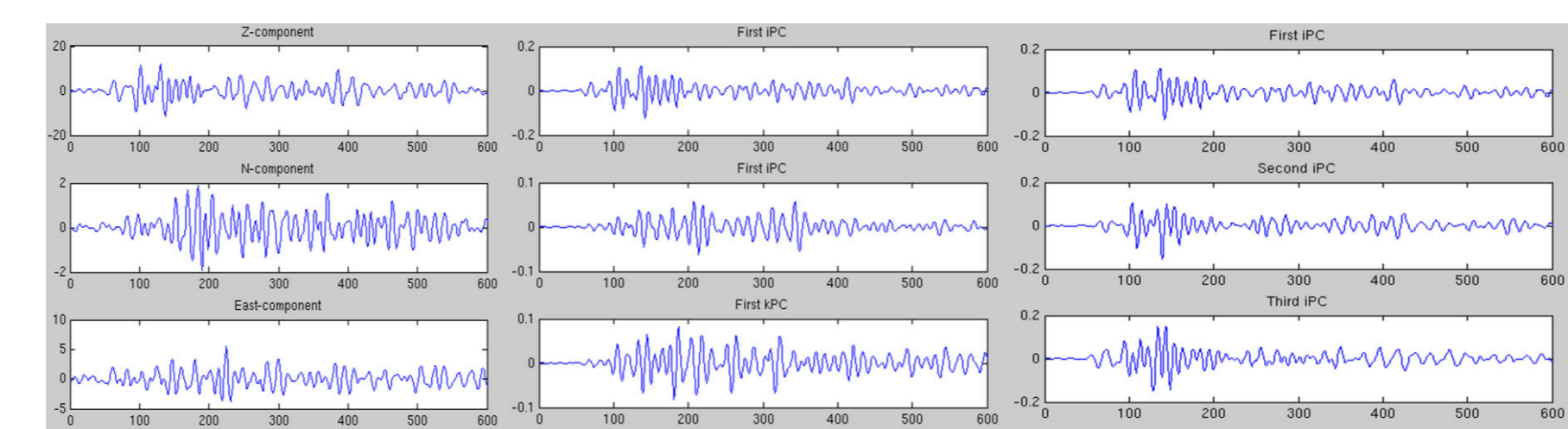
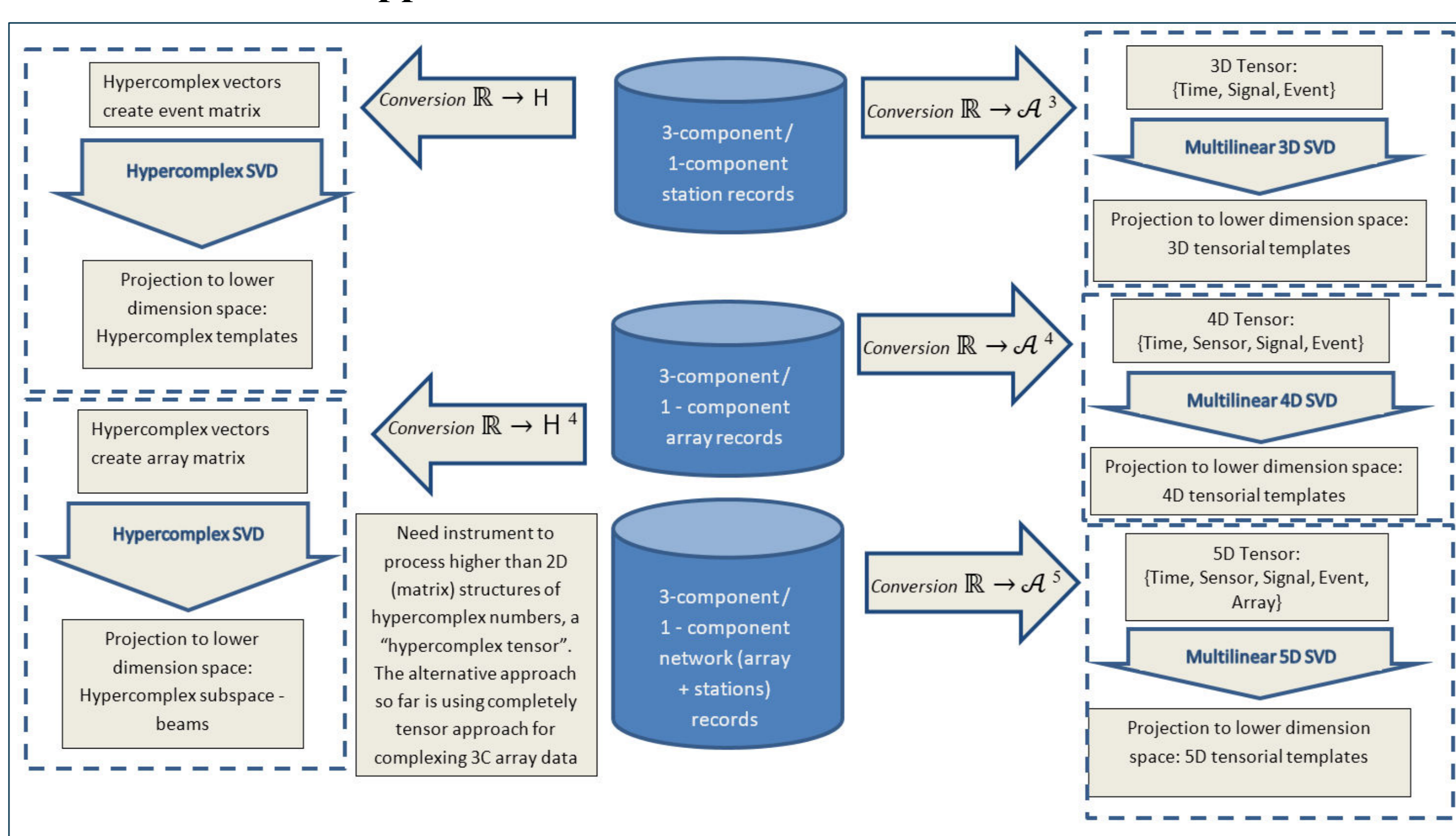
$$\mathbf{A} = \sum_{n=1}^r \mathbf{u}_n \mathbf{v}_n^c \sigma_n \quad \text{where } \mathbf{u}_n \text{ are the left singular vectors (columns of } \mathbf{U}) \text{ and } \mathbf{v}_n \text{ the right singular vectors (columns of } \mathbf{V}). \sigma \text{ are the real singular values. This expression shows that the SVDQ, as does the SVD in real and complex case, decomposes the matrix into a sum of } r \text{ rank-1 quaternion matrices (Nicolas Le Bihan, Jerome Mars, 2002).}$$

These were the definitions allowing approaching to building multidimensional seismic master events based on subspace model, or using multidimensional Principal Component Analysis. We follow Sangwine and Bihan, 2008 algorithm. This method bidiagonalizes **A** to a real bidiagonal matrix **B** using quaternion Householder transformations and a simple recursive algorithm based on that of Golub and Kahan, 1965. The SVD of B, computed with any available SVD algorithm for real matrices, followed by the multiplication of each of two quaternion matrices by a real matrix, yields the SVD of A. We use the Sangwine - Bihan qtfm hypercomplex MATLAB/OCTAVE library, version 2.3.0 2016-1-27, GNU GPL, which also overloads standard MATLAB classes with quaternion and octonion implementations. We also follow the Sangwine, Eli and Moxey, 2001, Electronics Letters 6th December 2001 Vol. 37 No. 25 for quaternion phase correlation implementation for color image processing, where

$$r(m, n) = \mathbf{F}^{-R} \{ R_R(m, n) \} \quad \text{and} \quad p(m, n) = \mathbf{F}^{-R} \left\{ \begin{matrix} R_R(m, n) \\ \overline{R_R(m, n)} \end{matrix} \right\} \quad \text{cross- and phase-correlations, and}$$

$$R_R(m, n) = \overline{F_L(v, u)} G_{Rl}(v, u) + \overline{F_{-L}(v, u)} G_{Rl}(v, u) \quad \text{is a quaternion cross spectrum (see figure to the right)}$$

Generalized approach to multidimensional master event construction



Example quaternion principle components. Left: input 3C seismogram, Center: pure quaternion i, j, and k components of first principal component, Right: first complex quaternion component of first 3 principle components.

Hypercomplex cross-correlation issues.

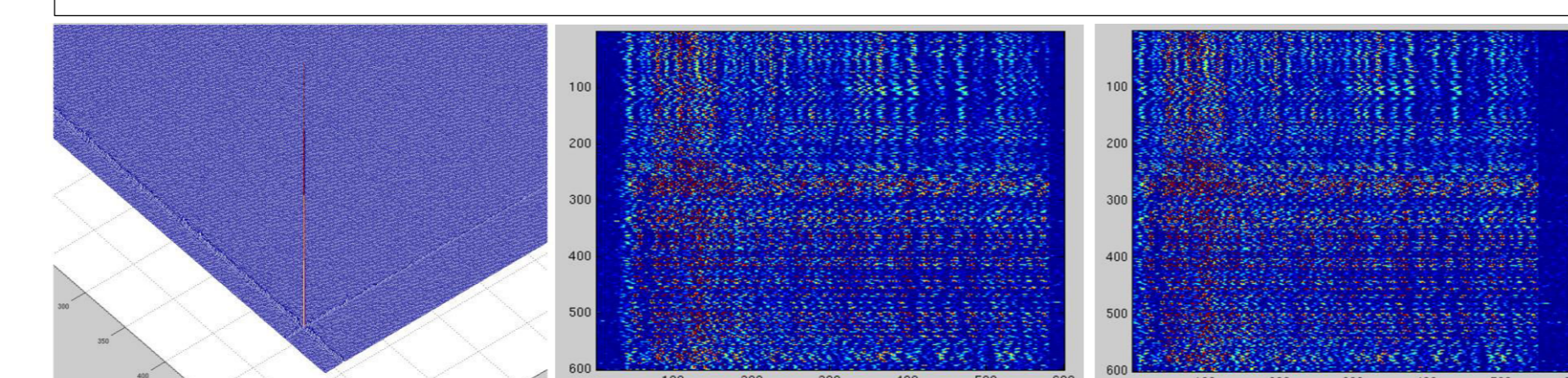
There are certain problems applying straight correlation approach as in case of real vector. For instance event for pure input quaternion vector (i, j, k) matrix, singular vectors are not pure in general case (s, i, j, k). Also, the scalar product of quaternion arguments is quaternionic, and so the angle between the quaternion vectors. So we needed to find some appropriate and intuitively understandable measure for event comparison. To come up with it, we studied the properties of real and complex quaternion parts relationships for different quaternionic products (scalar, cross, and convolution) tested on 3-component seismic quarry blast data (please also see our poster on Jordan quarry blast processing T-3.7-P12). We performed the quaternion dimensionality reduction through Q-SVD matrices truncation. Input data is a quaternion representation of 3C seismic records. The test is a cross-correlation between the:

1. Input Q-data vs input Q-data, with the search of the signal best correlating with the other signals.
2. Reduced Q-data after SVD-based restoration vs input Q-data.

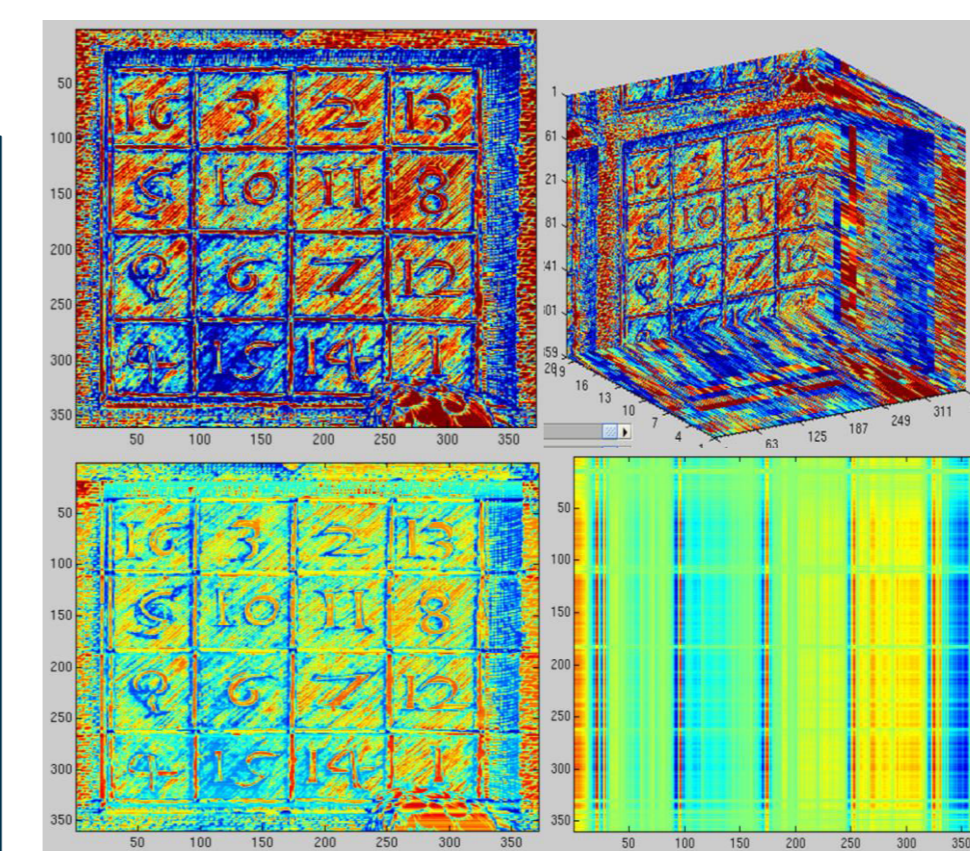
The measures were taken:

1. A norm of a quaternion cross-correlation coefficient (Q-CC) taken in its classical form.
2. A scalar or pure quaternion of the Q-CC (s(Q-CC), or norm (pure(Q-CC))
3. An angle in a quaternion space of the Q-CC

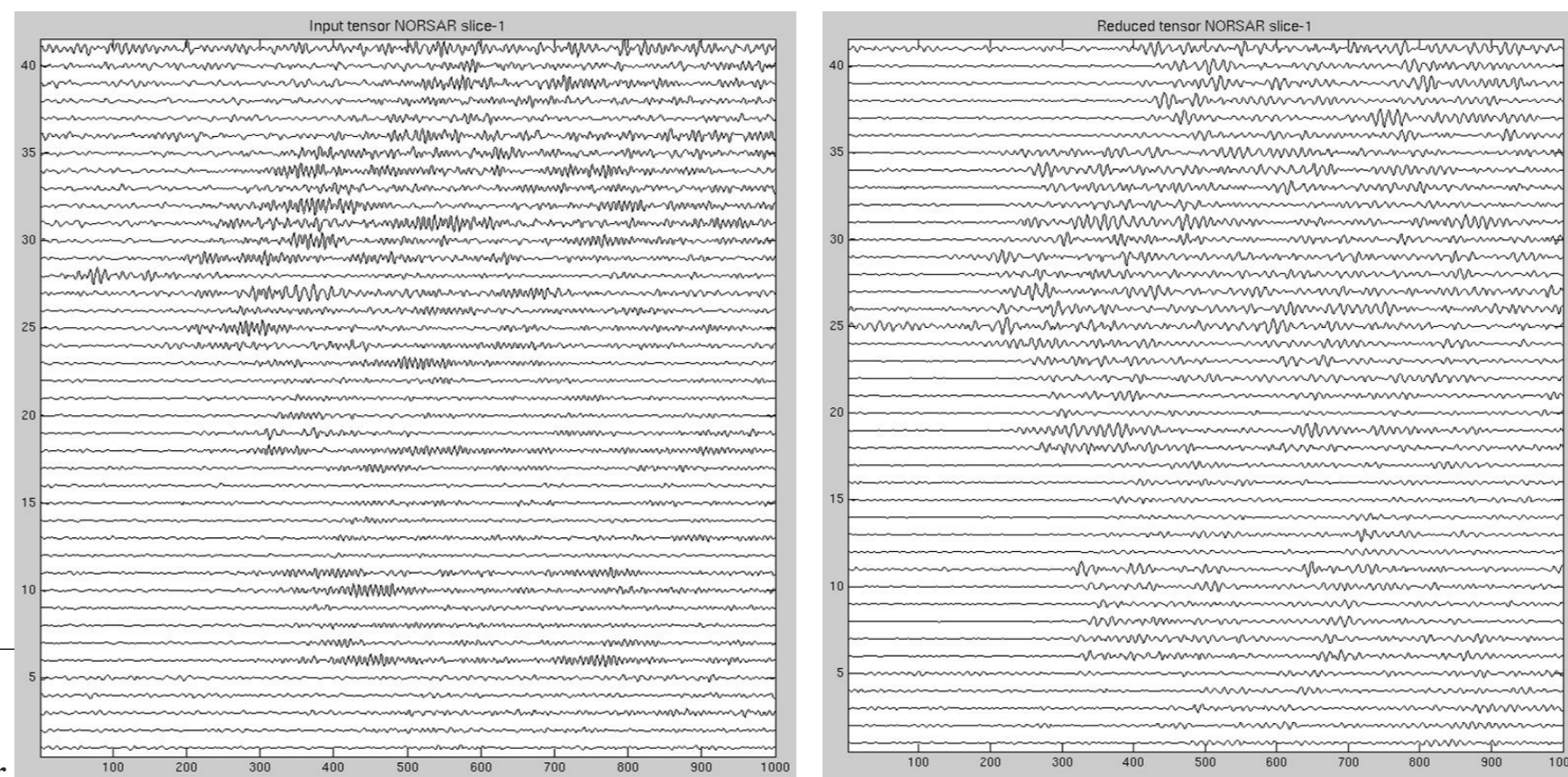
We found that the s(Q-CC) (scalar part of the quaternionic correlation coefficient, see formula to the left) appears to be the most sensitive measure of the Q-CC for this kind of data and this kind of test. The gain of this measure vs best raw signal CC is about 1.5. The gain based on full Q-CC is 1.24 and the gain based on the angle measure is 1.16 for the input set of 500 3C seismograms of 15 seconds length.



Quaternion phase correlation between two shifted (in time-event space) color images of 600x3C seismograms. A pick on correlation surface clearly indicates the shifts.



Top: Visualizing the tensor factor reduction through multilinear SVD. A standard "detail" 359x371 image (up) is x-y shifted 20 times and packed as a tensor, then the last slice is extracted with tensor decomposition (4D tensor processing). Below: same (single) image is decomposed (3D tensor) and reduced according to the eigenvalue meaningfulness. It corresponds to 3D filtering, thus extracting needed structure (noise or signal in case of seismic applications, see figures to the right).



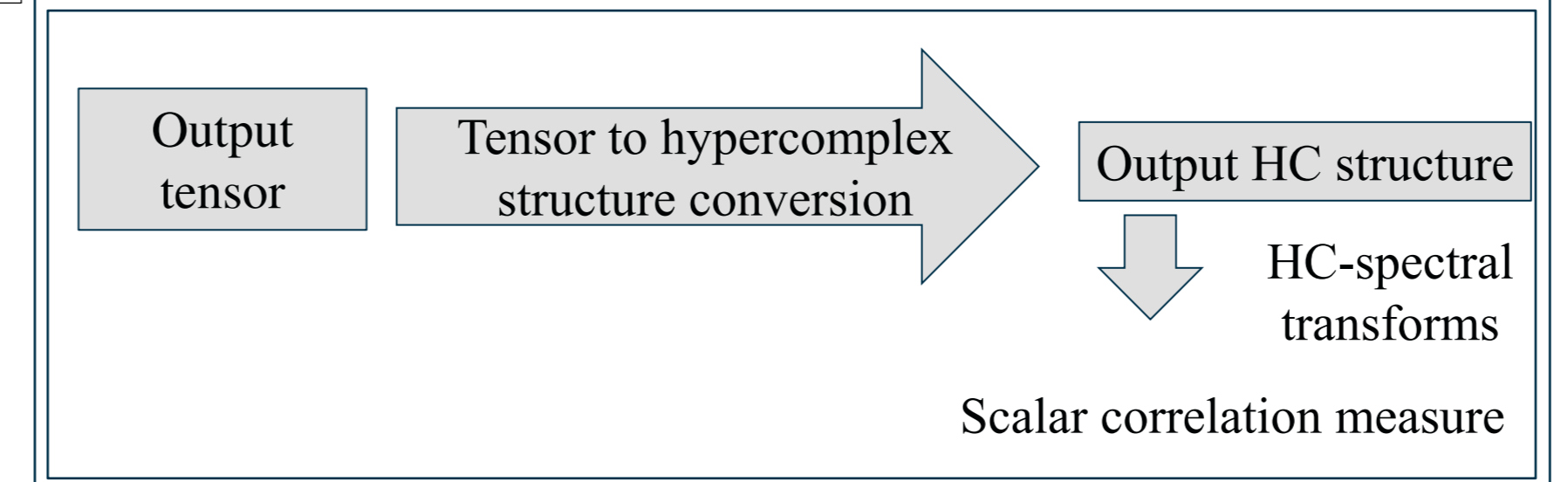
A tensor containing records of N events of length M from the arrays of M Z - sensors can be thought as a 3D cube sliced to N slices, and the reduced tensor - to R slices where R corresponds to the meaningful part of eigenvalue. 4D cube can represent the same of 3C array, or a network of L-arrays.

Observations

- Multidimensional eigen-structures are not inter-aligned, so straight eigen-images cannot be used for templates.
- There can be different approaches to the cross-correlation of multidimensional structures, we are still in a process of defining the best one compliant with nature of data and its dimensionality.

Approaches used:

1. To create eigen-image inheriting original timing (for example, for array having same time delays for central sensor), we use the reduced eigen-structure projection to original vector space.
2. Another approach is extracting single eigen-components and use them in conventional cross-correlation processing.
3. As it was shown for the quaternion structures, an angle based on norm and scalar product can be used to measure the divergence of the observed event from the designed master event. Same can be said for tensors but using Frobenius norms. However, in certain cases well-established mathematical apparatus of phase correlation developed for colour object processing can be used for multidimensional seismic structures as well.
4. We consider utilizing the hypercomplex cross-correlation approaches for tensor-based results in following manner:



The highlights of the mathematical formalism for tensor decomposition are given on our poster T3.7-P6 which describes the approach to construction of multidimensional templates for the recordings of mining explosions at IMS Scandinavian arrays. Here we will only highlight the approach to the *Tensor Subspace Projection for Dimensionality Reduction*. For any tensor

$$\mathcal{A} = \mathcal{S} \times_1 U^{(1)} \times_2 U^{(2)} \dots \times_N U^{(N)}, \text{ or}$$

$$\mathbf{A}_{(n)} = \mathbf{U}^{(n)} \cdot \mathcal{S}_{(n)} \cdot (\mathbf{U}^{(n+1)}) \dots \mathbf{U}^{(n+2)} \dots \mathbf{U}^{(N)} \otimes \mathbf{U}^{(1)} \otimes \mathbf{U}^{(2)} \dots \otimes \mathbf{U}^{(n-1)}$$

For typical image and video tensor objects although the corresponding tensor space is of high dimensionality, tensor objects typically are embedded in a lower dimensional tensor subspace (or manifold), in analogy to the vectorized face image embedding problem where vector image inputs reside in a low-dimensional subspace of the original input space (M. Turk, et al 1991). Thus, it is possible to find a tensor subspace that captures most of the variation in the input tensor objects and it can be used to extract features for recognition and classification applications. To achieve this objective, $P_n < I_n$ orthonormal basis vectors (principle axes) of the n -mode linear space \mathbb{R}^{I_n} are sought for each mode n and a tensor subspace $\mathbb{R}^{P_1} \otimes \mathbb{R}^{P_2} \dots \otimes \mathbb{R}^{P_N}$ is formed by these linear subspaces.

The MPCA objective is to define a multilinear transformation

$$\{\bar{\mathbf{U}}^{(n)} \in \mathbb{R}^{I_n \times P_n}, n = 1, \dots, N\} \text{ that maps the original tensor space } \mathbb{R}^{I_1} \otimes \mathbb{R}^{I_2} \dots \otimes \mathbb{R}^{I_N} \text{ into a tensor subspace } \mathbb{R}^{P_1} \otimes \mathbb{R}^{P_2} \dots \otimes \mathbb{R}^{P_N} \text{ (with } P_n < I_n \text{ for } n = 1, \dots, N):$$

$$Y_M = X_M \times_1 \bar{\mathbf{U}}^{(1)T} \times_2 \bar{\mathbf{U}}^{(2)T} \dots \times_N \bar{\mathbf{U}}^{(N)T}, m = 1, \dots, M, \text{ such that}$$

$\{Y_M \in \mathbb{R}^{P_1} \otimes \mathbb{R}^{P_2} \dots \otimes \mathbb{R}^{P_N}, m = 1, \dots, M\}$ captures most of the variations observed in the original tensor objects, assuming that these variations are measured by the total tensor scatter.

In other words, the MPCA objective is the determination of the N projection matrices

$$\{\bar{\mathbf{U}}^{(n)} \in \mathbb{R}^{I_n \times P_n}, n = 1, \dots, N\} \text{ that maximize the total tensor scatter } \Psi_Y:$$

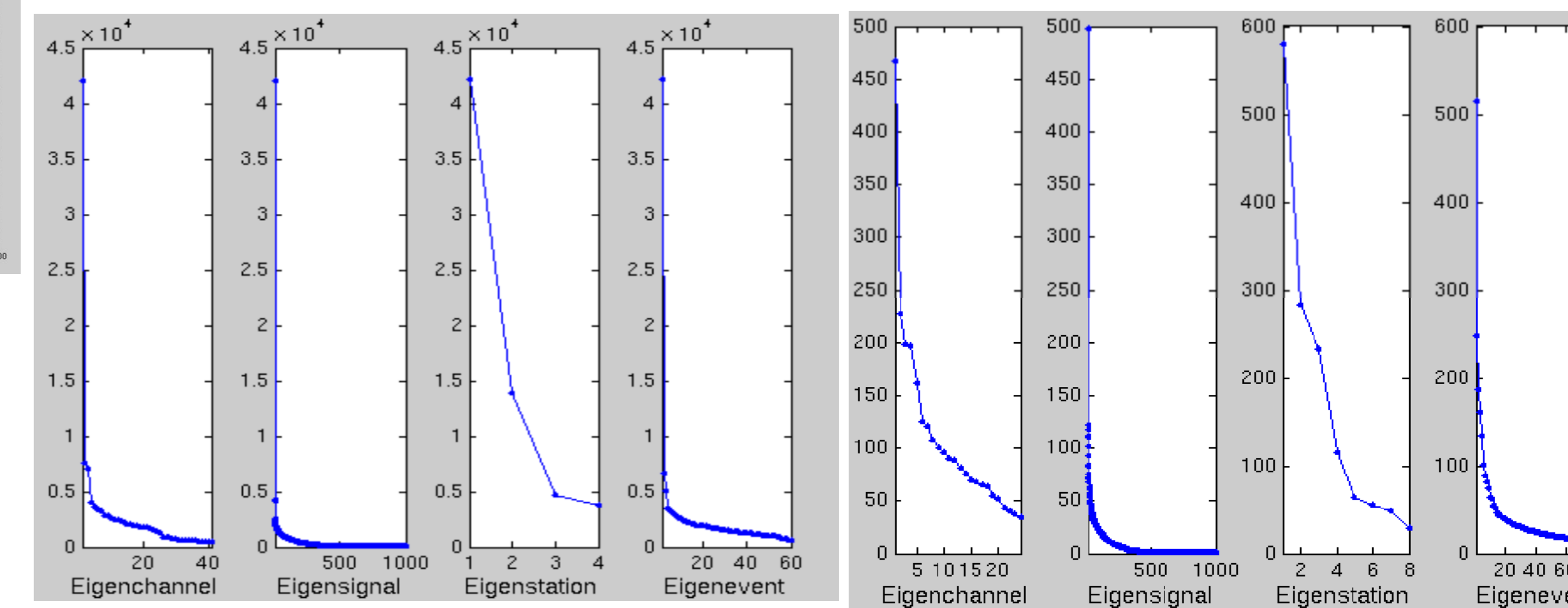
$$\{\bar{\mathbf{U}}^{(n)}, n = 1, \dots, N\} = \text{arg}_{\bar{\mathbf{U}}^{(1)}, \bar{\mathbf{U}}^{(2)}, \dots, \bar{\mathbf{U}}^{(N)}} \max \Psi_Y$$

Here, the dimensionality P_n for each mode is assumed to be known or predetermined (Lu, Plataniotis, and Venetsanopoulos, 2006).

We perform a dimensionality reduction through the truncation of U and \mathcal{S} terms and building the restored tensor \mathcal{A}_{red} through the block term decomposition (BTD) which approximates a tensor by a sum of low multilinear rank terms.

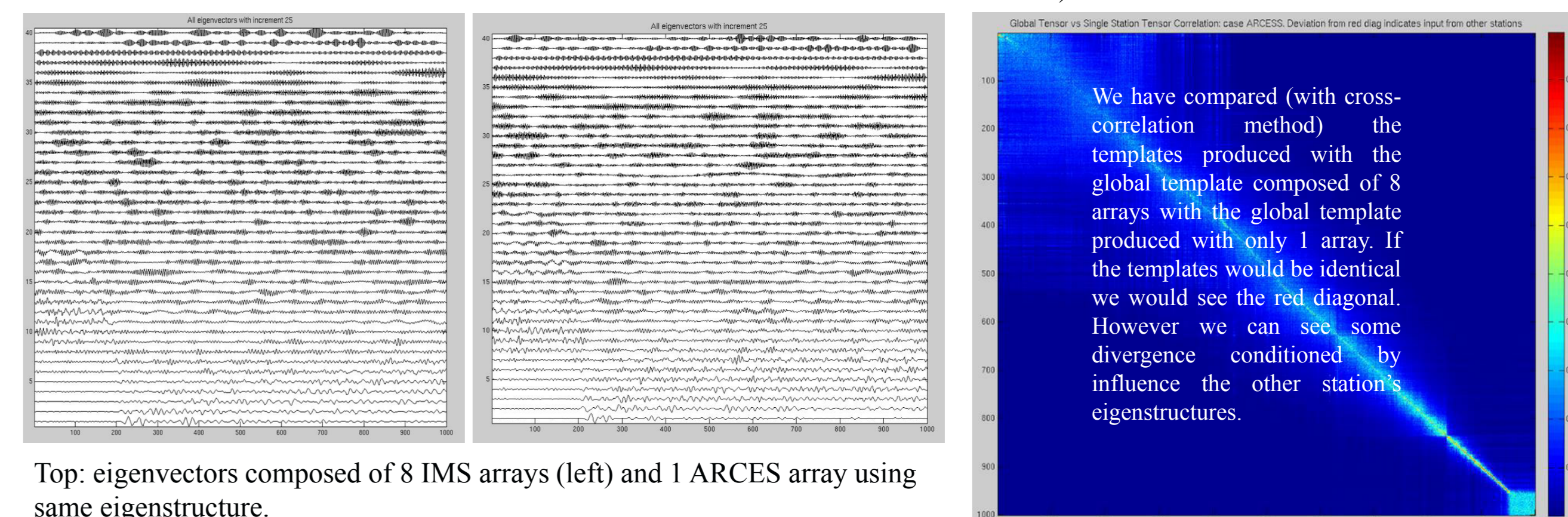
$$\mathcal{A}_{red} = \sum_{r=1}^R \mathcal{S}^{(r)} \times_1 U^{(r,1)} \times_2 U^{(r,2)} \times_3 \dots \times_N U^{(r,N)}$$

The P_n can be determined for any tensor mode, taking into account the most meaningful parts of eigenstructure:



Global eigenstructure of the tensor composed by the 8-station IMS subnet composed of stations AKASG; FINES; MKAR; NVAR; TXAR; USRK; WRA; ZALV. Global seismicity is studied.

Regional eigenstructure of the tensor composed by the 4-station IMS regional subnet composed of stations ARCES, NORSAR, FINES and HFS. Regional seismicity (mining explosions at Keruna and Aitik) is studied.



Top: eigenvectors composed of 8 IMS arrays (left) and 1 ARCES array using same eigenstructure.

Implementation. Our processing tool uses TensorToolbox, TensorReg and TensorLab toolboxes as a core (see below) providing the tensor SVD (High Order, or HO SVD or Multilinear, or MSVD).

A TensorToolbox software by T. Kolda (Sandia National Lab).

The HO Discrete Cosine Transform based reduction and the dimensionality reduction based on tensor interpolation is also based on TensorReg software by Zhou, 2013.

The TensorLab 3.0, 2016 toolbox is by Vervliet N., Debals O., Sorber L., Van Barel M. and De Lathauwer.

2DPCA discussed in (Lu et al, 2006) is also tested in this work through the TensorReg as well. We are not discussing the results in this presentation since we could not obtain good enough performance with this method.