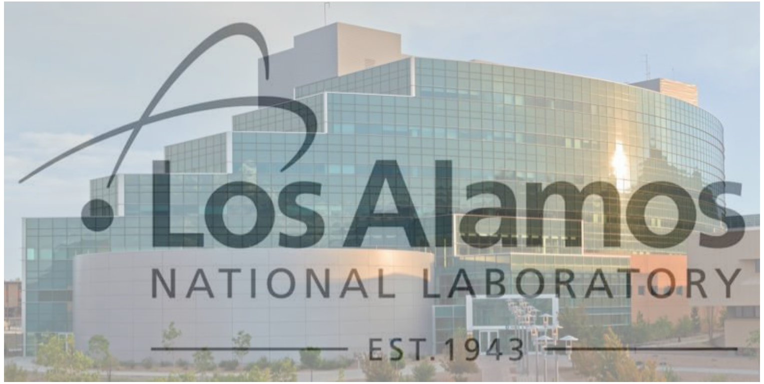
LA-UR-16-29291

View in Slide-Show Mode for Animations

Developing and Validating Path-Dependent Uncertainty Estimates for Use with the Regional Seismic Travel Time (RSTT) Model



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T1.2-O1 Solid Earth Structure

June 29, 2017

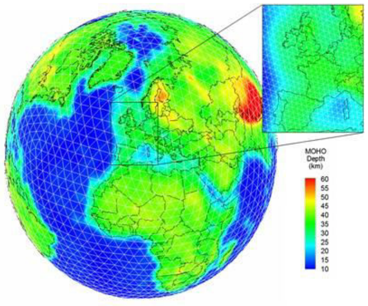
Sponsoring Organizations:
U.S. Department of Energy, National Nuclear Security Administration, Office of Defense Nuclear Nonproliferation R&D
U.S. Department of Energy, National Nuclear Security Administration, Office of Nonproliferation and Arms Control

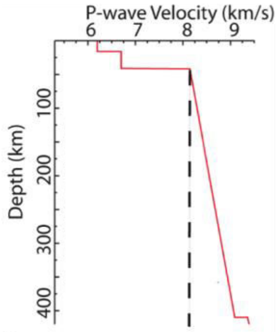
Introduction and Outline

- **Regional Seismic Travel Time (RSTT) code and models (current version rstt201404um): http://www.sandia.gov/rstt**
  - Fast way to accurately predict travel times for regional (< 15-17 deg) phases for location algorithms (Pn, Sn, Pg, Lg).
- **RSTT Summary**
- **Developing an Error Model for Travel-time Uncertainty (all regional, showing Pn, Pg)**
  - 1D Distance dependent
  - 2D Path dependent
    - *Why it's necessary*
    - *Construction*
    - *Implementation*
- **Location testing (REB Pn arrivals):**
  - Models
  - Stations
  - Uncertainty estimates
- **Conclusions and Future Work**

RTSS Summary

- **Global tessellation**
  - Velocity profiles (velocity vs. depth) at each node are interpolated to render a 3-dimensional crust that overlies a laterally variable representation of upper mantle velocity.
  - The mantle velocity parameterization is specified by the velocity at the crust-mantle boundary and a linear velocity gradient as a function of depth. (2.5D)
  - RSTT is a type of **GeoTess model** (with some additions): **http://www.sandia.gov/geotess**
- **Calculation**
  - Approximation of travel time (~1 ms computation)
  - Accounts for diving rays in a velocity gradient (Zhao and Xie, 1993)
    - *c term related to gradient and Earth curvature*
  - Pg and Lg calculation is similar to the Pn/Sn calculation but without the diving-ray correction.





Modified from Myers et al. (2010)

$$TT = \sum_{i=1}^{N} d_i s_i + \quad + \quad +$$

d= path increment immediately below crust/mantle boundary.
s=1/velocity
α= source-side portion of ray.
β= receiver-side portion of ray

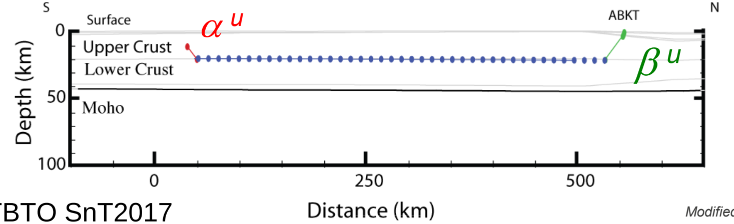
$$= \frac{c^2 X_m^3}{24 V_0}$$

see Myers et al. (2010)

Pg/Lg

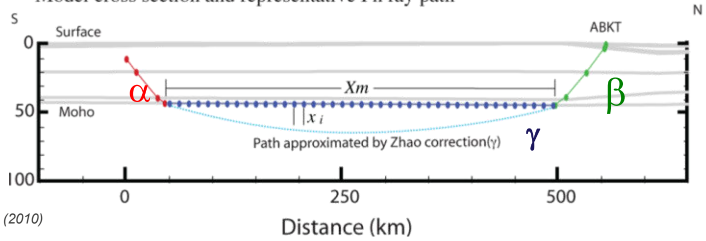
$$T = \sum_{i=1}^{N} x_i s_i + \quad ^u + \quad ^u$$

Model cross section and representative Pg/Lg ray path



Pn/Sn

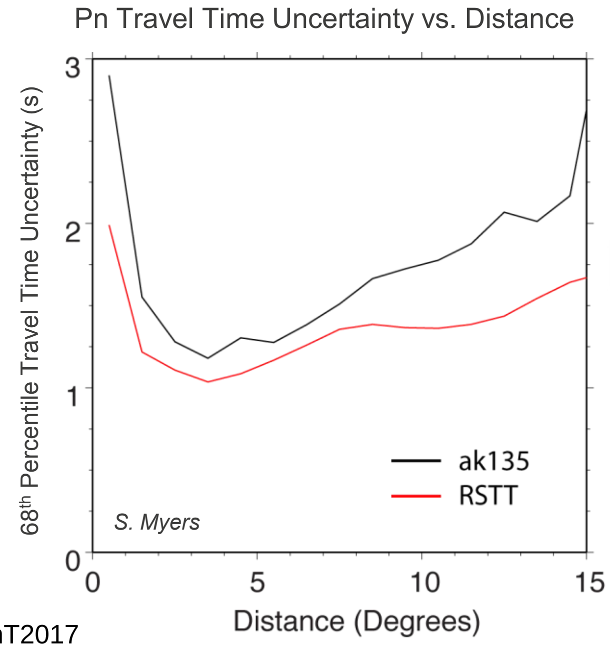
Model cross section and representative Pn ray path

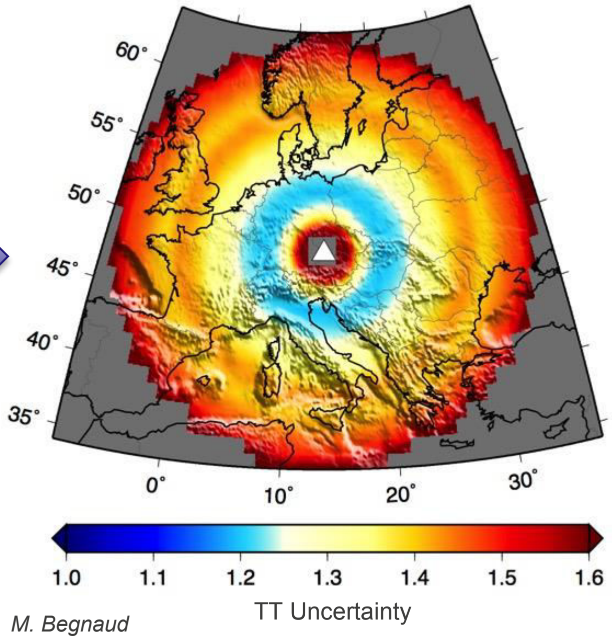


Modified from Myers et al. (2010)

1D Distance-Dependent Uncertainty

- **Traditional travel time uncertainty (aka "model" error) is calculated from binning travel time residuals by distance and finding a percentile or standard deviation**
  - For higher-dimensional models (e.g., RSTT – 2.5D), treats all areas/stations the same





2D Path-Dependent Uncertainty

- **For a 2/2.5D model like RSTT, we want to have a path-dependent uncertainty determination with a similar dimension as the model itself.**
  - Accounts for spatial variations in empirical data which could lead to bias in the model depending on the specific ray path.
  - Variations in residuals among model nodes allow for separation into “model” and “random” error components.
  - The tomographic model is global in extent, but there are many areas where there are no ray paths. The uncertainty calculations must account for that in some way.
- **Others have calculated full 3D model covariance matrix (e.g., SALSA3D: Hipp et al., 2012; Ballard et al., 2016)**
  - Works well, but computationally expensive
  - Goal for RSTT is to have the general user be able to estimate a path-dependent travel time prediction uncertainty **on-the-fly** (e.g., NDC in a box)
  - *For a 2.5D model at regional distances, can we achieve the prediction uncertainty metric (i.e., coverage ellipse contains ground-truth event at certain percentage)?*

Constructing a 2D Error Model for RSTT

- **Develop an error model based on physical representation**
  (o = source, i = path/receiver)

E = *model error*
ε = *measurement/random error*
μ = *bias*

$$t_i = t_o + (T_{oi} + \quad_T + E_T) + \quad_i$$

- **Same general model can be used for multiple estimates (observations) of apparent slowness local to node *n* (i.e., tomography)**

For any node *n* along path *i*:

$$s_i = (S_n + \quad_S + E_S) + \quad_i$$

$$s_i \quad S_n = \quad_S + E_S + \quad_i$$

for Pn, Sn phases

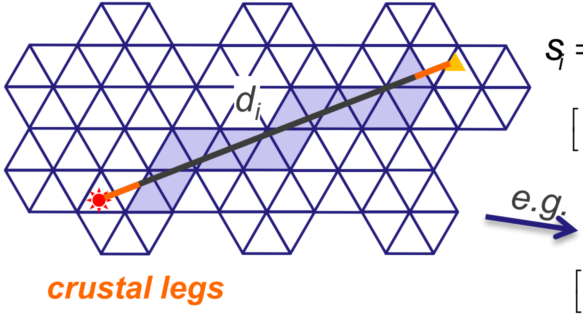
$$s_i = \frac{(t_i \quad t_o) \quad (T_{oi,s} + T_{oi,r} + T_{oi,g})}{d_i}$$

- **Using a Two-Way Random Effects model to estimate node bias and variance components from residual error:**
  Bias(μ) Var(E_S) = τ^2 Var(ε_i) = σ^2

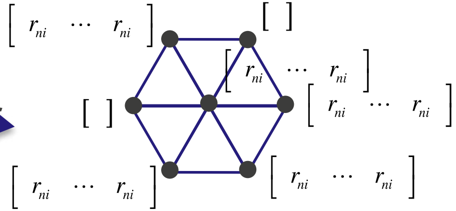
t_i = arrival time (s)
t_o = origin time (s)
T_{oi} = calculated total travel time for path (s)
$T_{oi,s}$ = source crustal leg calculated travel time for path (s)
$T_{oi,r}$ = receiver crustal leg calculated travel time for path (s)
$T_{oi,g}$ = gradient component of calculated travel time for path (s)
s_i = observed apparent slowness for path *i* (s/km)
d_i = mantle leg path length (km)
S_n = slowness (inverse velocity) at node *n* (s/km)

Collect Slowness Residuals for Each Ray Path

- **After tomography, slowness residuals for a phase are assigned to all “touched” nodes of the model along each ray path.**



$$s_i = \frac{(t_i \quad t_o) \quad (T_{oi,s} + T_{oi,r} + T_{oi,g})}{d_i}$$



For each node *n* on a triangle that is touched by the mantle ray path *i*,

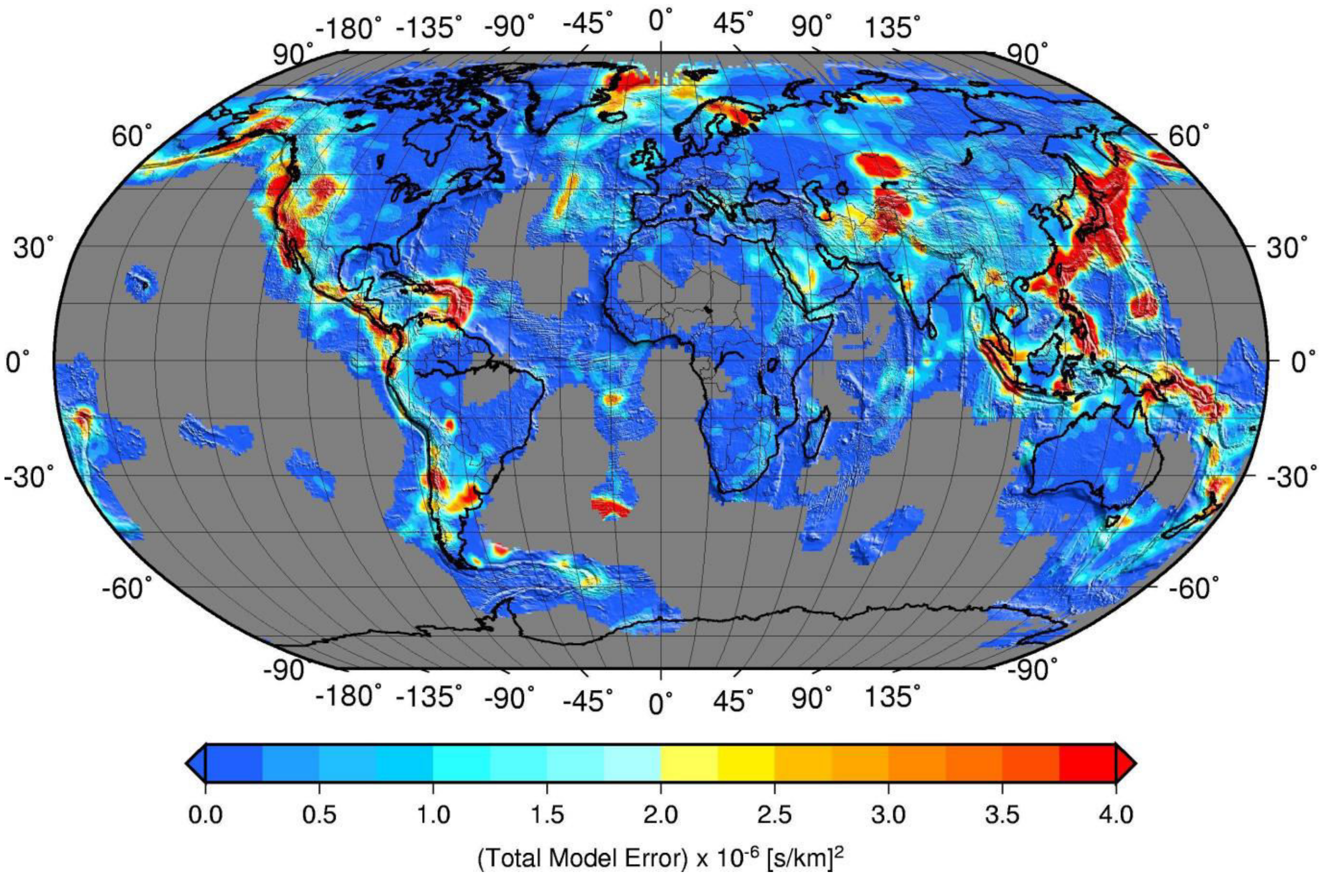
$$r_{ni} = s_i \qquad S_n$$

- **Residuals are gathered for each node in the model and combined with residuals for each surrounding node**
  - <u>Two-Way Random Effects model</u>
    - *Random (Hexagon, Node)*
    - *Nested: Nodes are part of Hexagons*
      - Model error (τ^2) has components from Hexagons and Nodes
    - *Minimum of 2 nodes in collection*
      - Assign bias and variances to the center node

Collecting Slowness Residuals and Factors Affecting Uncertainty Values

- **Collections of slowness residuals are determined in two ways:**
  - (1) With all available data
  - (2) Binned by source-receiver distance (1° bins to 15°)
- **Bias (μ) and variances (τ^2–model, σ^2–random) are calculated only where there are available data for a phase (e.g., Pn, Pg)**
- **Factors that can affect the uncertainty values:**
  - Crustal model
  - Inadequacy of the model resolution (e.g., complex regions)
  - Problems with physical assumptions
  - Data issues

Total Model Error (τ^2) Grid, All Data: Pn



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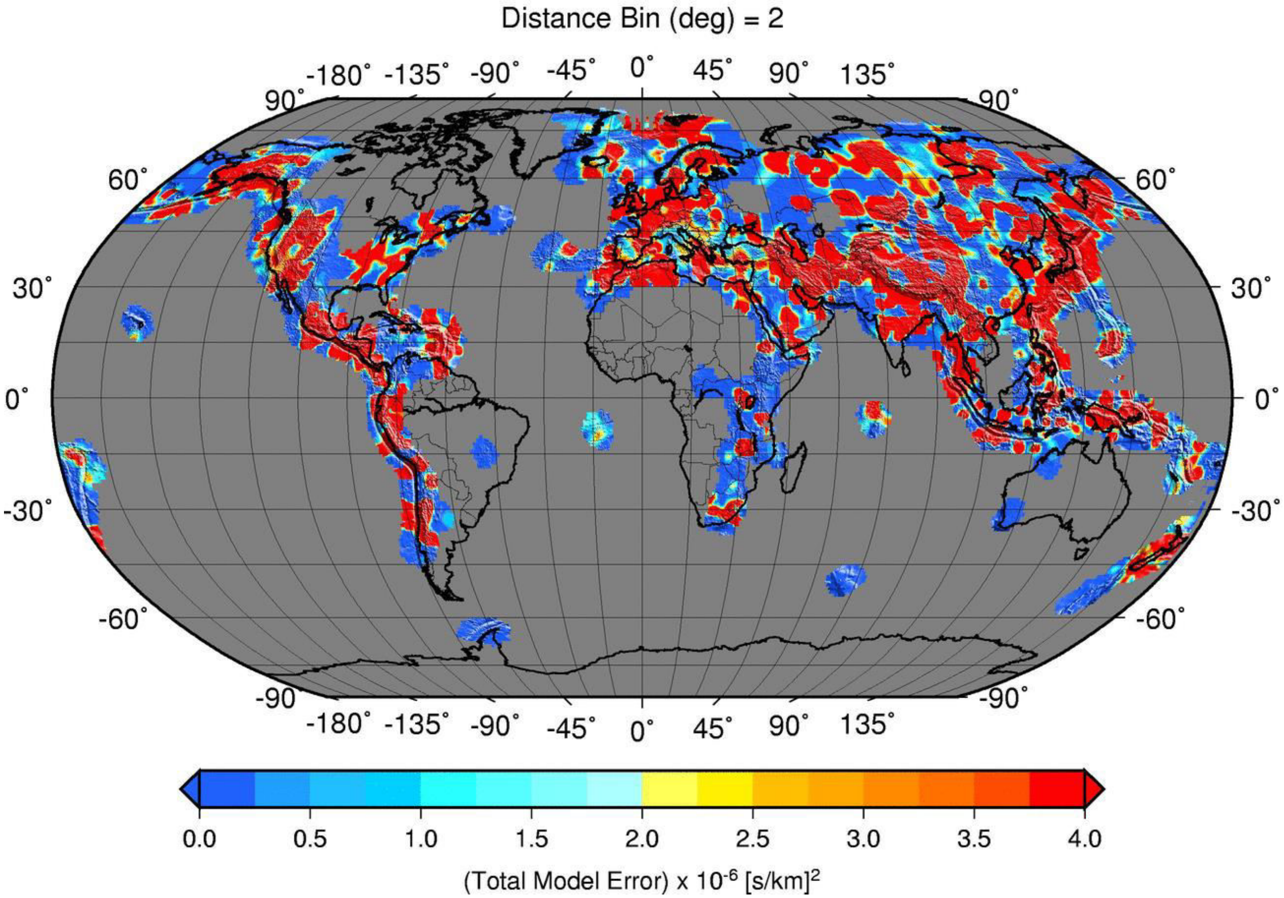
Total Model Error (τ^2), Distance Bins: Pn



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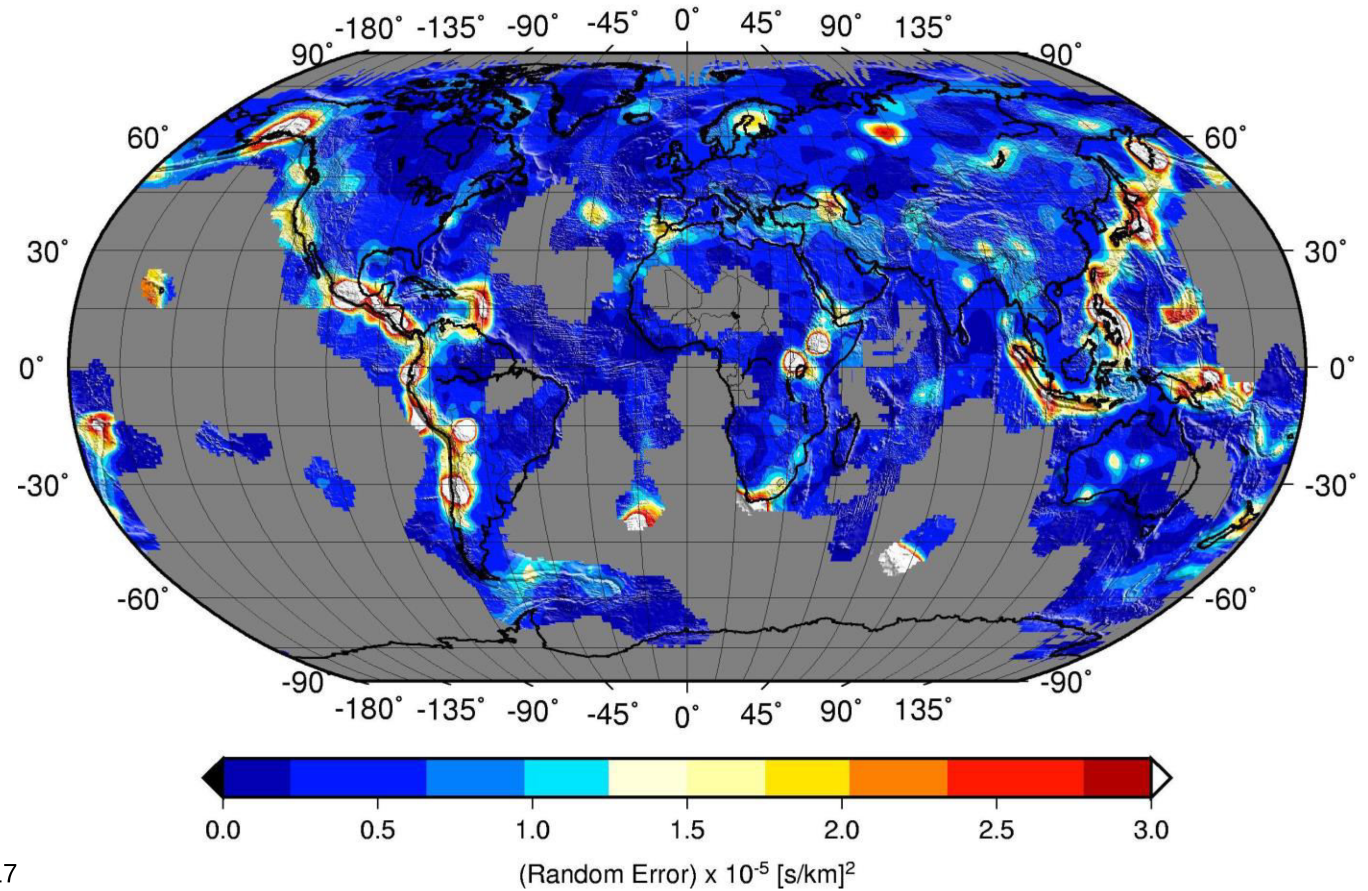
Random Error (σ^2) Grid, All Data: Pn



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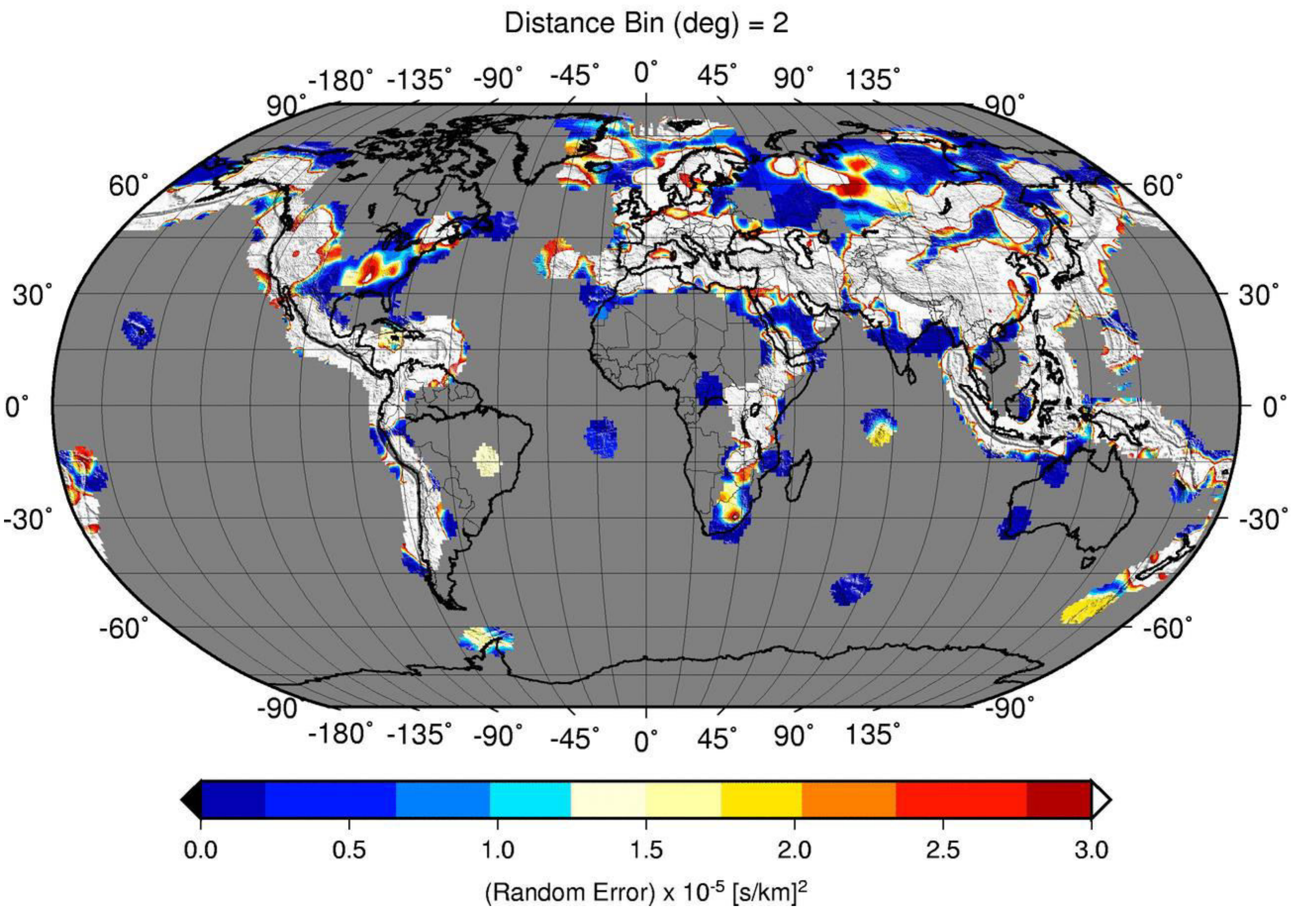
Random Error (σ^2), Distance Bins: Pn



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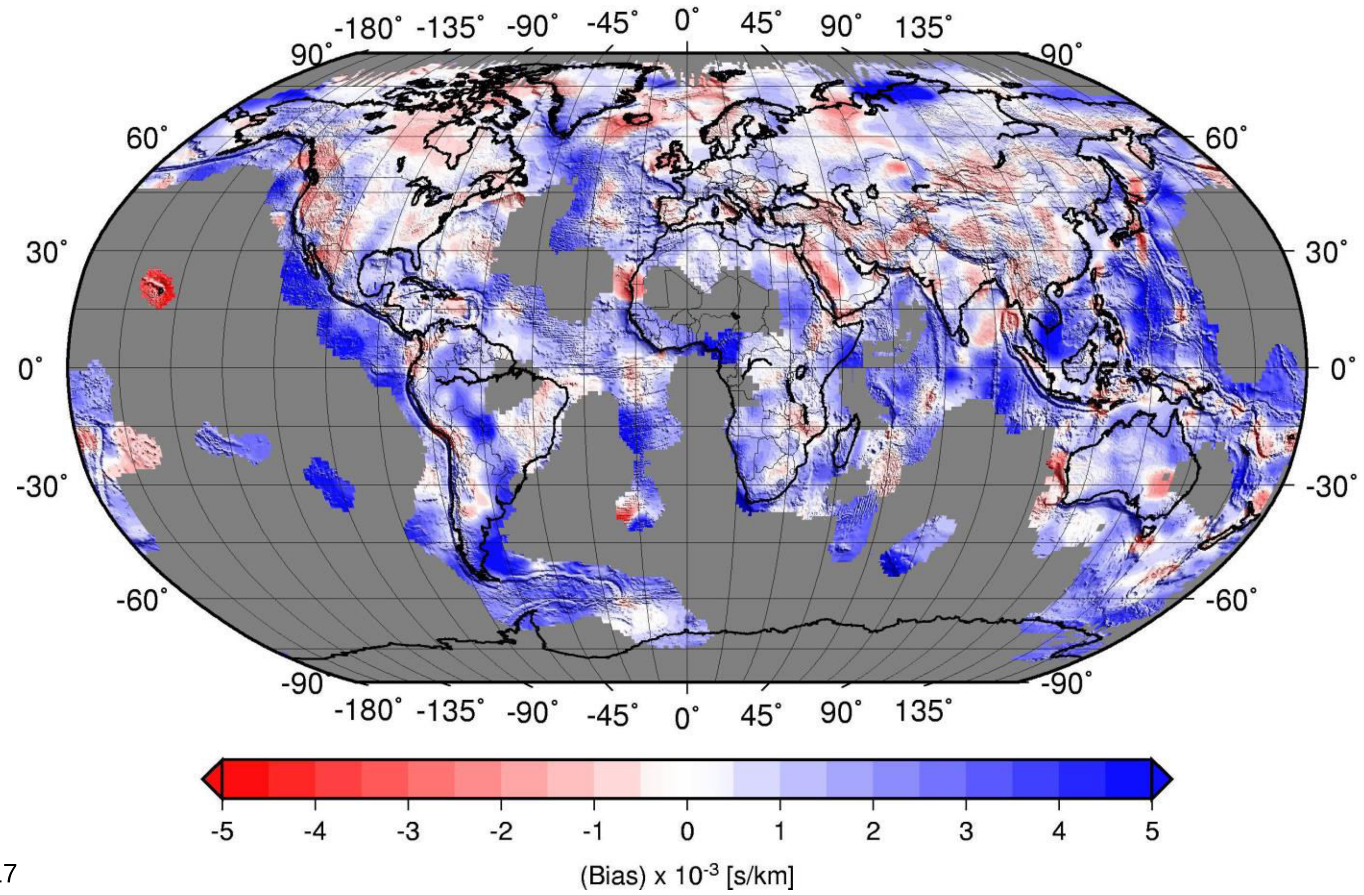
Bias (μ) Grid, All Data: Pn



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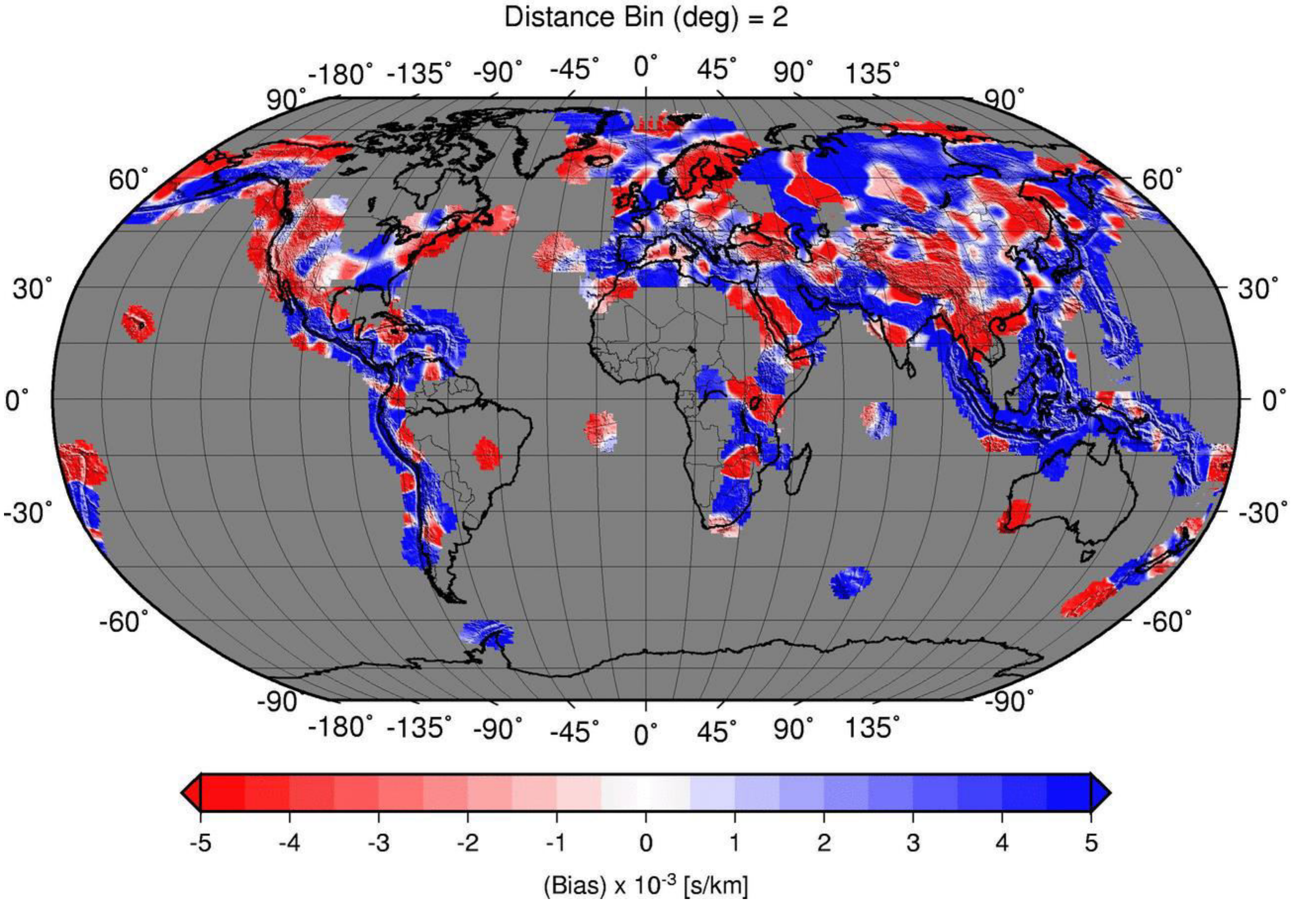
Bias(μ), Distance Bins: Pn



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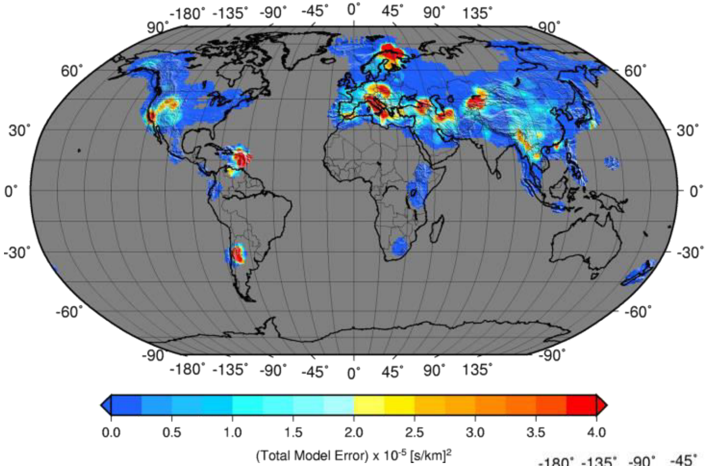
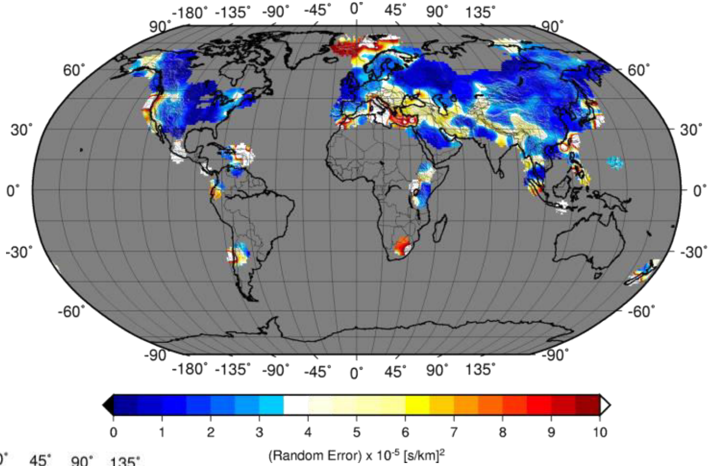
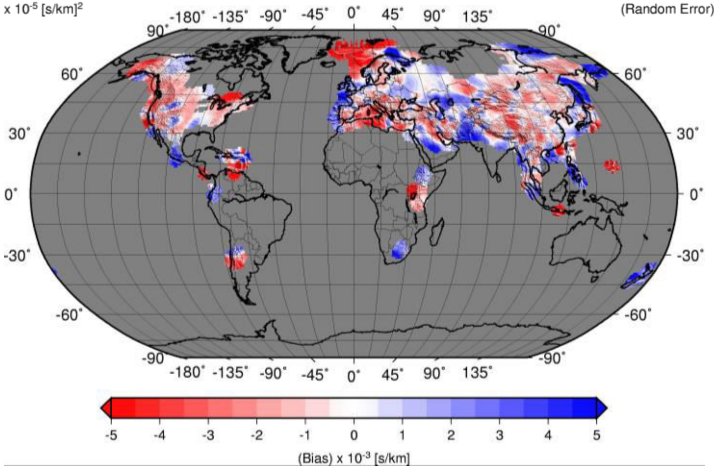
Total Model Error (τ^2), Random Error (σ^2), and Bias (μ) Grid -- All Data: Pg





Total Model Error (τ^2)

Random Error (σ^2)



Bias (μ)

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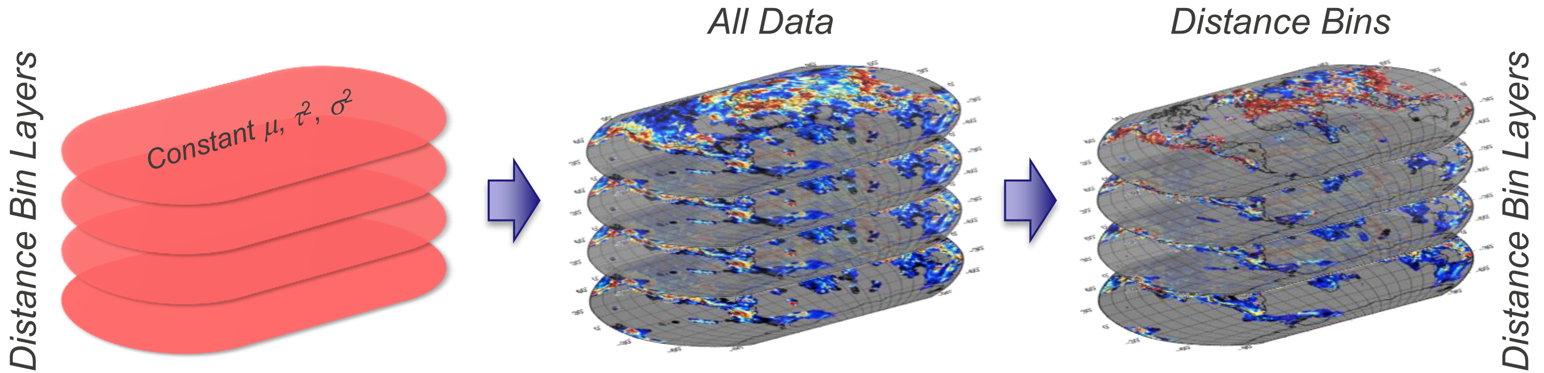
Building a Grid for Calculating Travel Time Uncertainty

- **Background bias/variances are estimated high (constant or boot-strap) to inform the user where the model is not constrained by data**
- **3D grid of sum squared error is built with distance bins as the Z-dimension** *(see figure below)*
  - Bias/Variance grid is built as **successive replacements**: (1) background, (2) all available, (3) distance bins
    - *End up with a complete grid of values, populated everywhere*



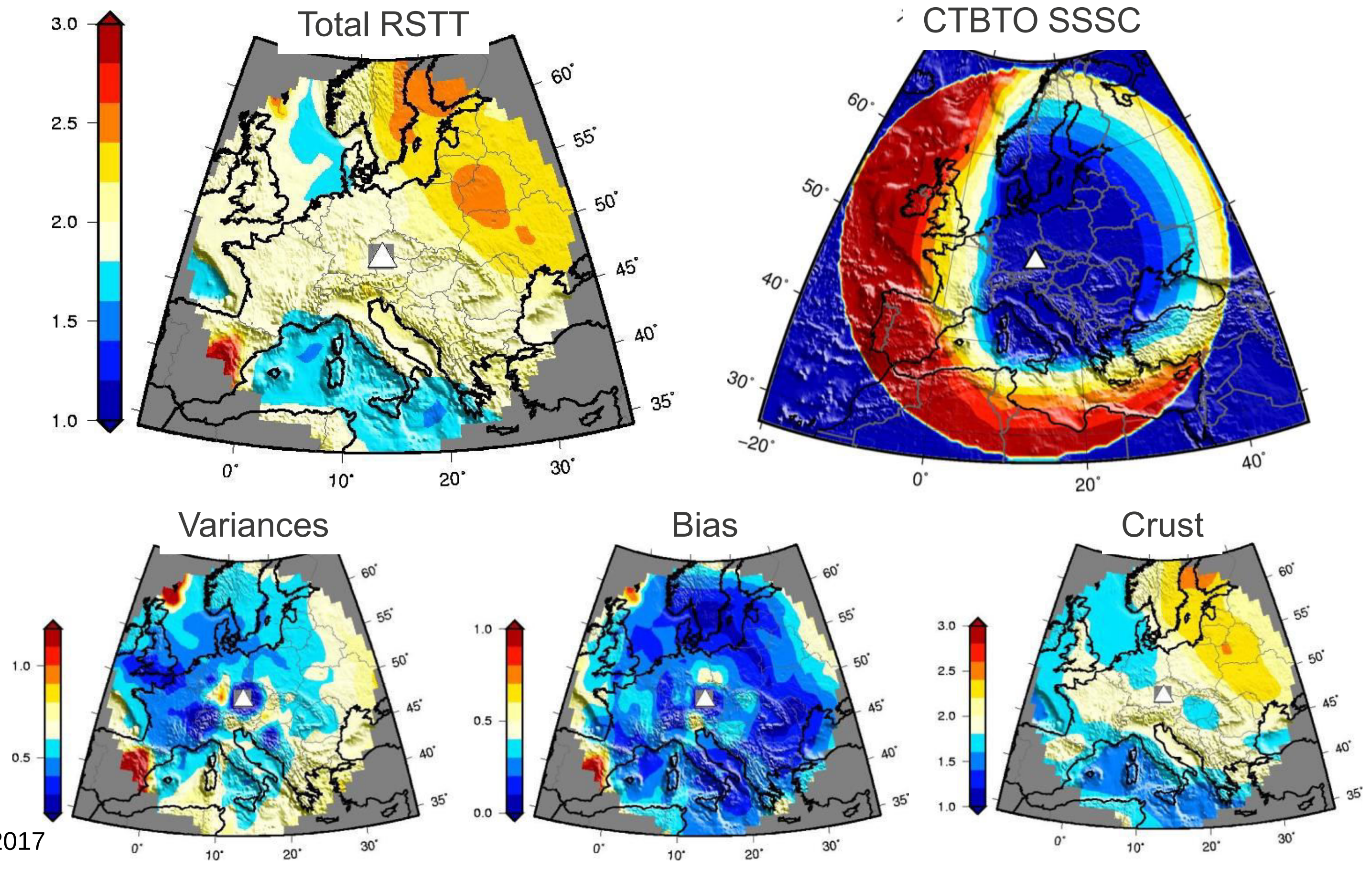
Calculating Travel Time Uncertainty

- **For a given ray path, gather the nodal weights for the tomography model (mantle, crust):** w_i, c_i
- **Using the source-receiver distance determine the bias/variance grid to use (interpolation between bins)**
  - For the nodes (*n*) along the mantle path, pull the bias/variances into a path-dependent model covariance matrix
    - *Assume non-zero covariance for node neighbors only*
- **Use a standard double-integral to calculate the path-dependent travel time uncertainty for the mantle**
- **Add a separate travel time uncertainty for the crustal portion**
  - Assume constant *a priori* crustal slowness uncertainty (ν^2) (based on variance of “touched” crustal nodes, *m*) (~15% average slowness)

$$P_{mantle} = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix} \begin{bmatrix} \mu_{11}^2 + \sigma_{11}^2 + \tau_{11}^2 & \tau_{12}^2 & \cdots & \tau_{1n}^2 \\ \tau_{12}^2 & \mu_{22}^2 + \sigma_{22}^2 + \tau_{22}^2 & \cdots & \tau_{2n}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \tau_{1n}^2 & \tau_{2n}^2 & \cdots & \mu_{nn}^2 + \sigma_{nn}^2 + \tau_{nn}^2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

$$\boxed{P_{crust} = \overset{m}{\underset{i=1}{}} \; {}^{2}c_i^{2}}$$

Pn Travel-time Uncertainty Surfaces (s): GERES



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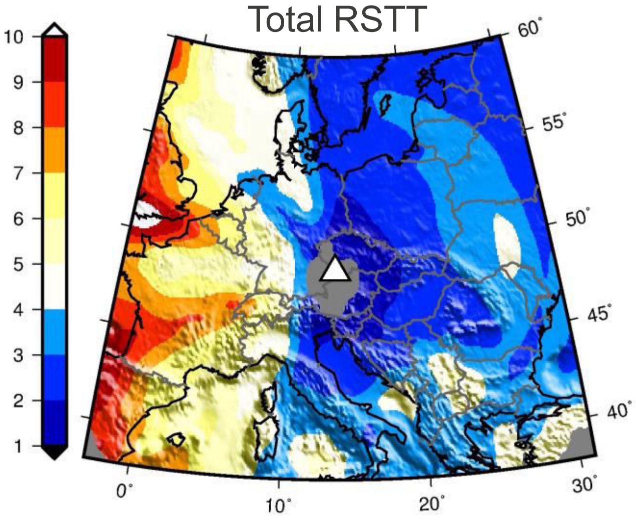
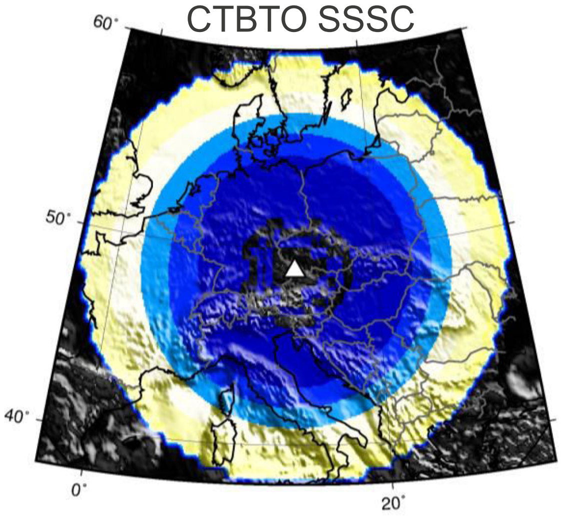
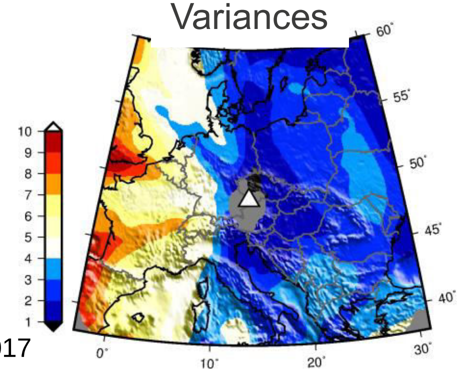
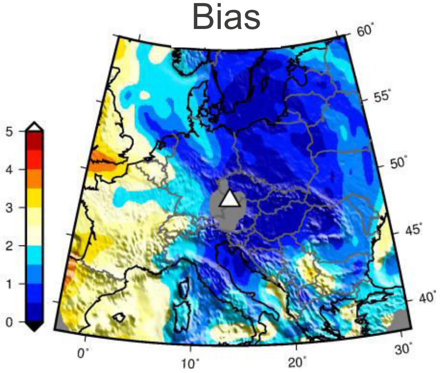
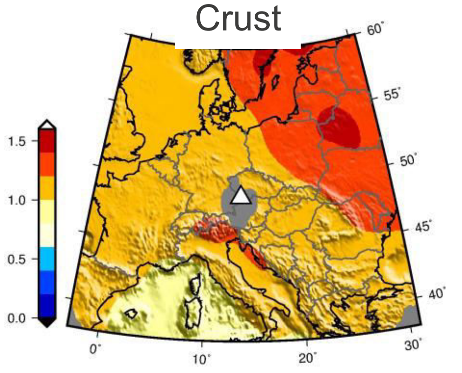
Pg Travel-time Uncertainty Surfaces (s): GERES











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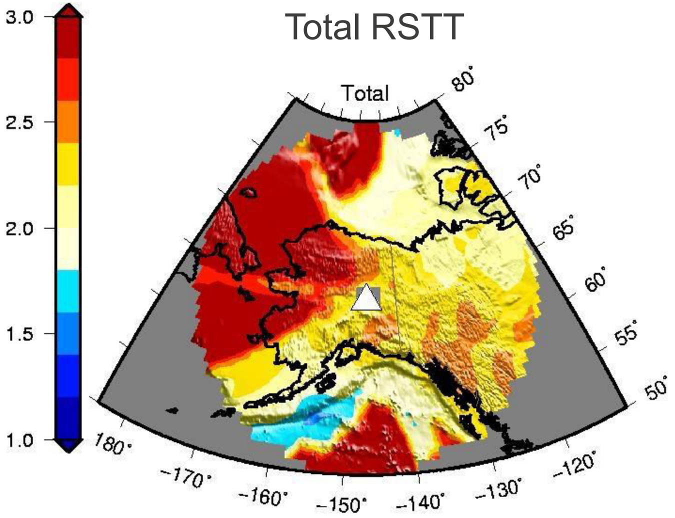
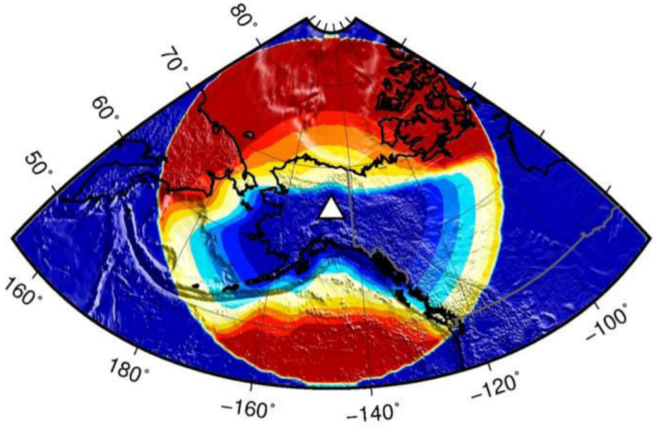
Pn Travel-time Uncertainty Surfaces (s): ILAR

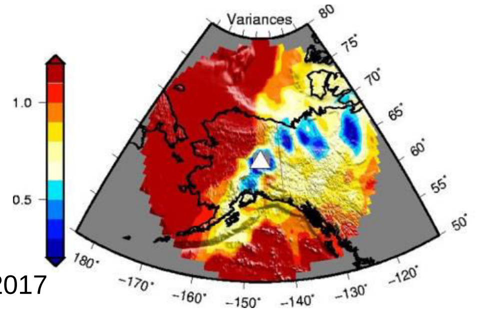
Total RSTT



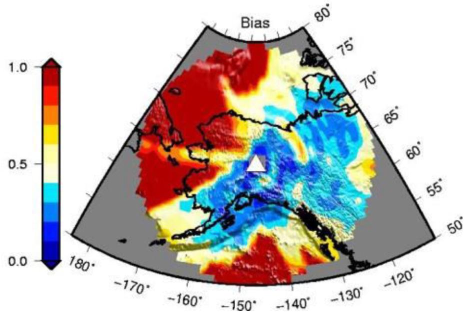
CTBTO SSSC



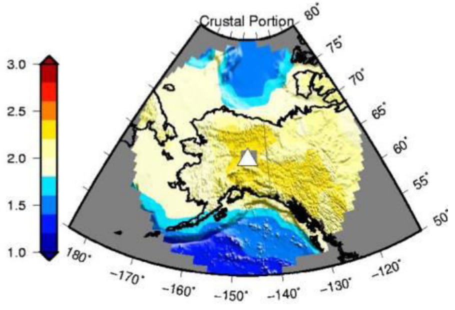
Variances



Bias



Crust



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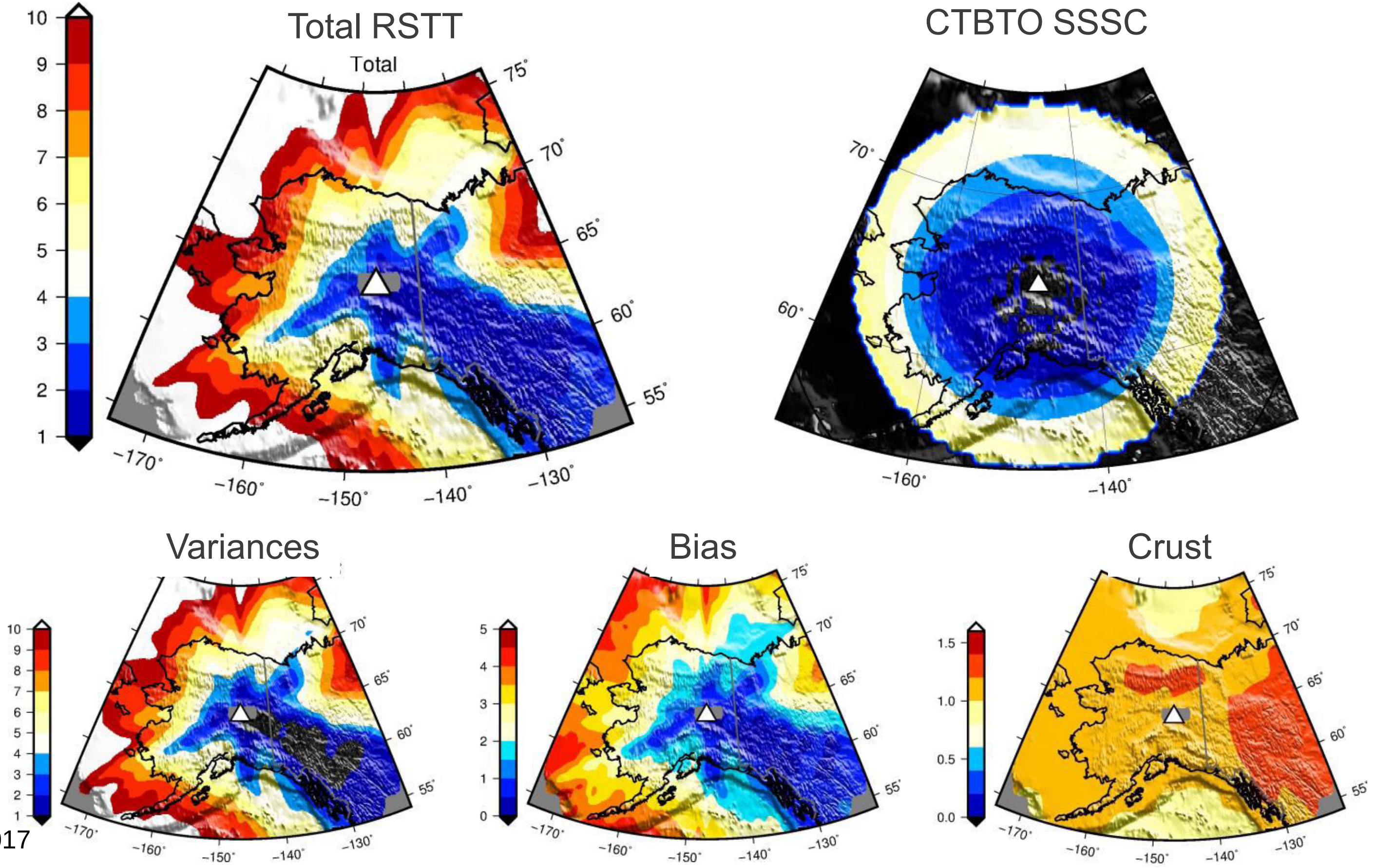
Pg Travel-time Uncertainty Surfaces (s): ILAR



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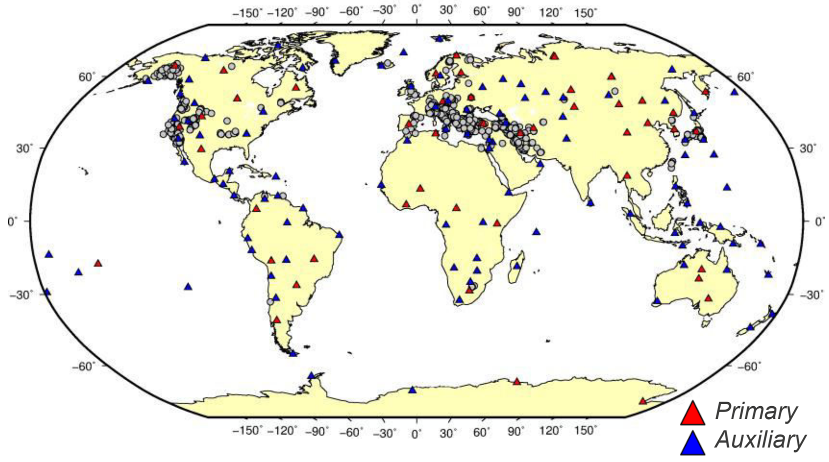
Validation of New Uncertainty Estimates

- ***Stations + Arrivals:***
  - All possible: IMS primary + auxiliary Pn arrivals (Pg not used in this test)
  - Only IMS stations with SSSCs
- ***Velocity Models Used***
  - ak135
  - CTBTO Source-specific Station Corrections (SSSCs) (converted to GeoTess grids)
  - *RSTT (on-the-fly)
  - *RSTT-SSSCs (SSSCs created from RSTT implementation -- Le Bras et al., 2013)
    - *Converted GeoTess grids (0.4 deg tessellation)*
    - *Correction to iasp91 at 10 km depth only*
    - *WGB45-WEG, Begnaud presented evaluation of this RSTT-SSSC implementation: "Evaluating RSTT-SSSC Extrapolations Beyond 15 degrees"*
  - SALSA3D (path-dependent uncertainty from full 3D covariance matrix) [Hipp et al., 2012; Ballard et al., 2016] (GeoTess grids)
- ***Uncertainty estimates:***
  - 1D distance-dependent or default
  - *New 2D path-dependent for RSTT-type models (uncertainty calculated at 0 depth)

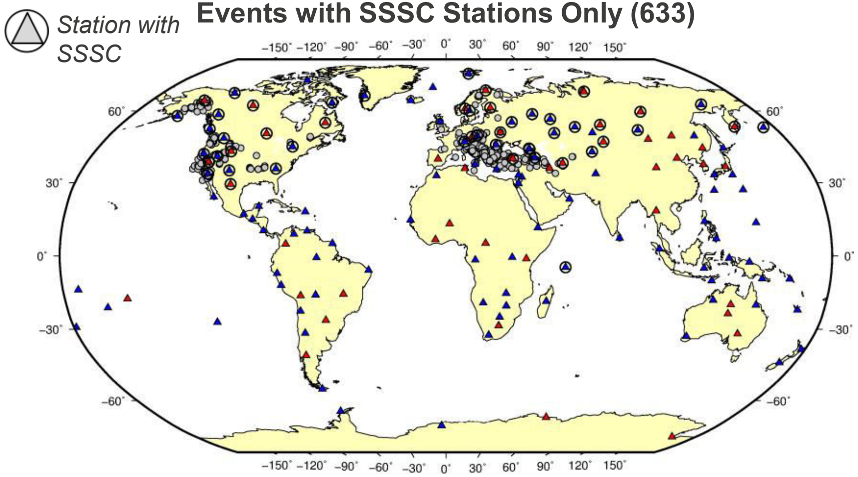
Validation Event Selection

- **Validation events chosen from set of GT5 or better events from the LANL Location Database (multi-catalog)**
  - Selected REB events from this GT set
  - Pn phases only (all available)
- **Station Sets:**
  - (1) Allowed any IMS primary or auxiliary station (921 events)
  - (2) Only stations with CTBTO SSSCs (633 events)
- **Fixed depth**
- **90% coverage ellipses**
- **RSTT-SSSC new uncertainty values calculated at 0 km depth**
  - travel-time corrections were calculated at 10 km

Events with Any IMS Station (922)



Events with SSSC Stations Only (633)



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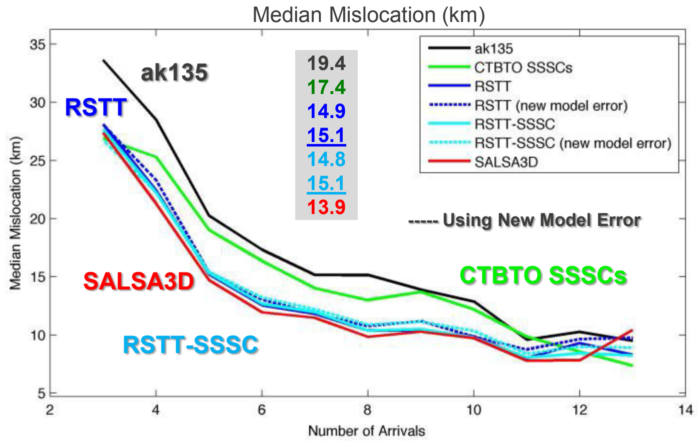
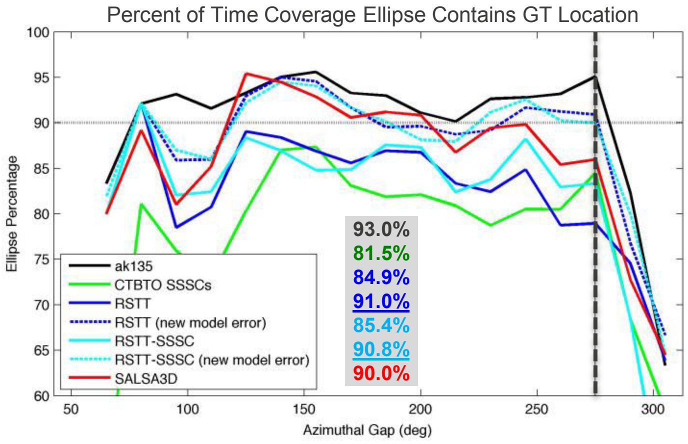
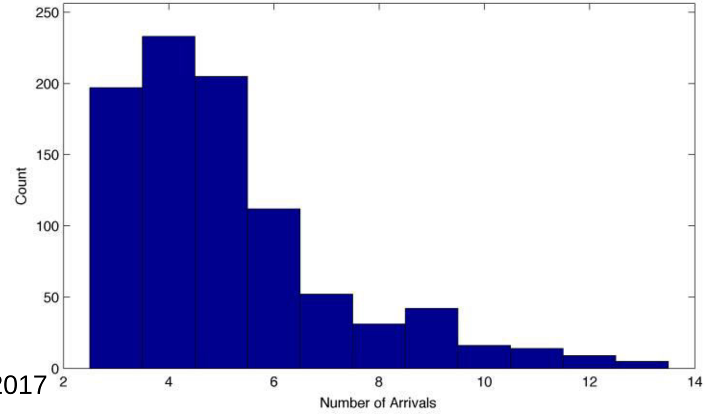
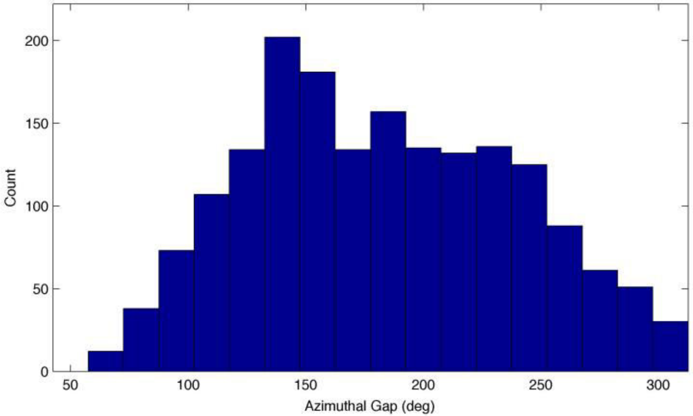
Location and 90% Coverage Error Ellipse Results: *GT5 or better Validation Events (921)* *All IDC Pn Phases*

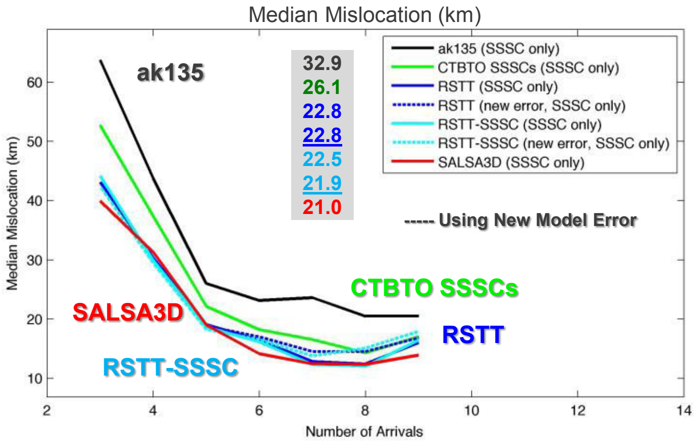


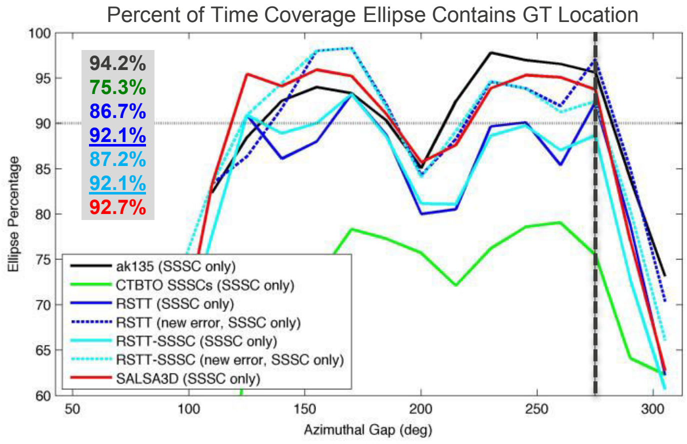


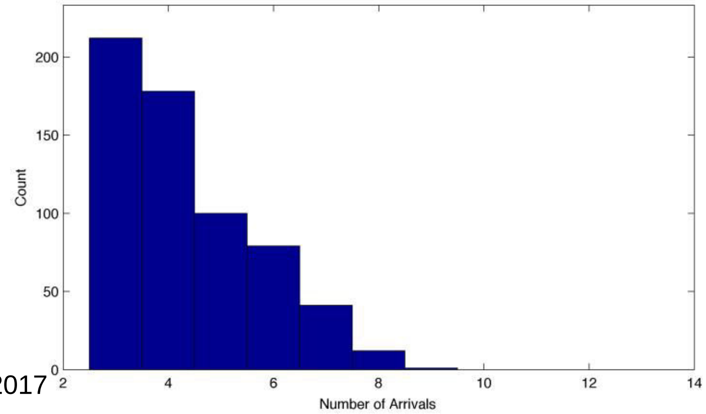


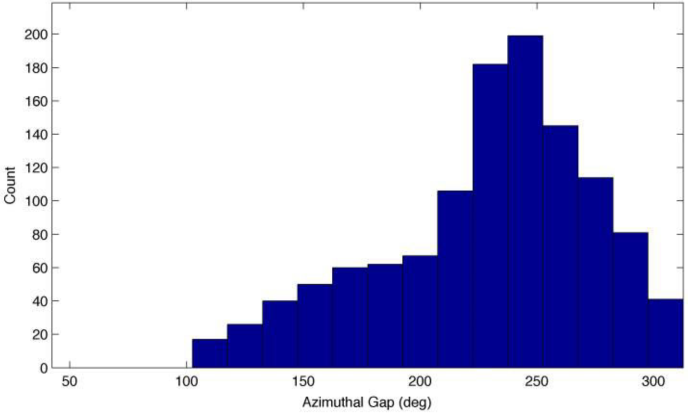


Location and 90% Coverage Error Ellipse Results: *GT5 or better Validation Events (633)* *Pn Phases from Stations with SSSCs Only*









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Conclusions

- **Can calculate a 2D, path-dependent travel time error for the RSTT model.**
- **Use apparent slowness residuals, collected at each "touched" node in the model, along with its surrounding neighbors.**
- **Use a Two-way Random Effects model for each node collection to estimate bias and model/random error.**
    - Model error tends to be higher in regions with high upper mantle gradient – consistent with previous travel time prediction tests.
- **Bias/Variance grids are built for each phase and distance bin, with high background error to tell the user when a path crosses into a model area that was not constrained by data.**
- **The travel time error for a specific path can be calculated <u>on-the-fly</u> using a covariance matrix with only surrounding nodes contributing to off-diagonal elements.**
- **Addition of a crustal portion of uncertainty is significant.**

Future Work

- Run synthetic tests (develop intuition about error process)
- Continue location testing with Pg, Sn, and Lg phases.
- Compare with calculating full model covariance matrix
- Apply IDC Analyst rules for time-defining in some way?
- Implement new uncertainty algorithm in RSTT public code.

References

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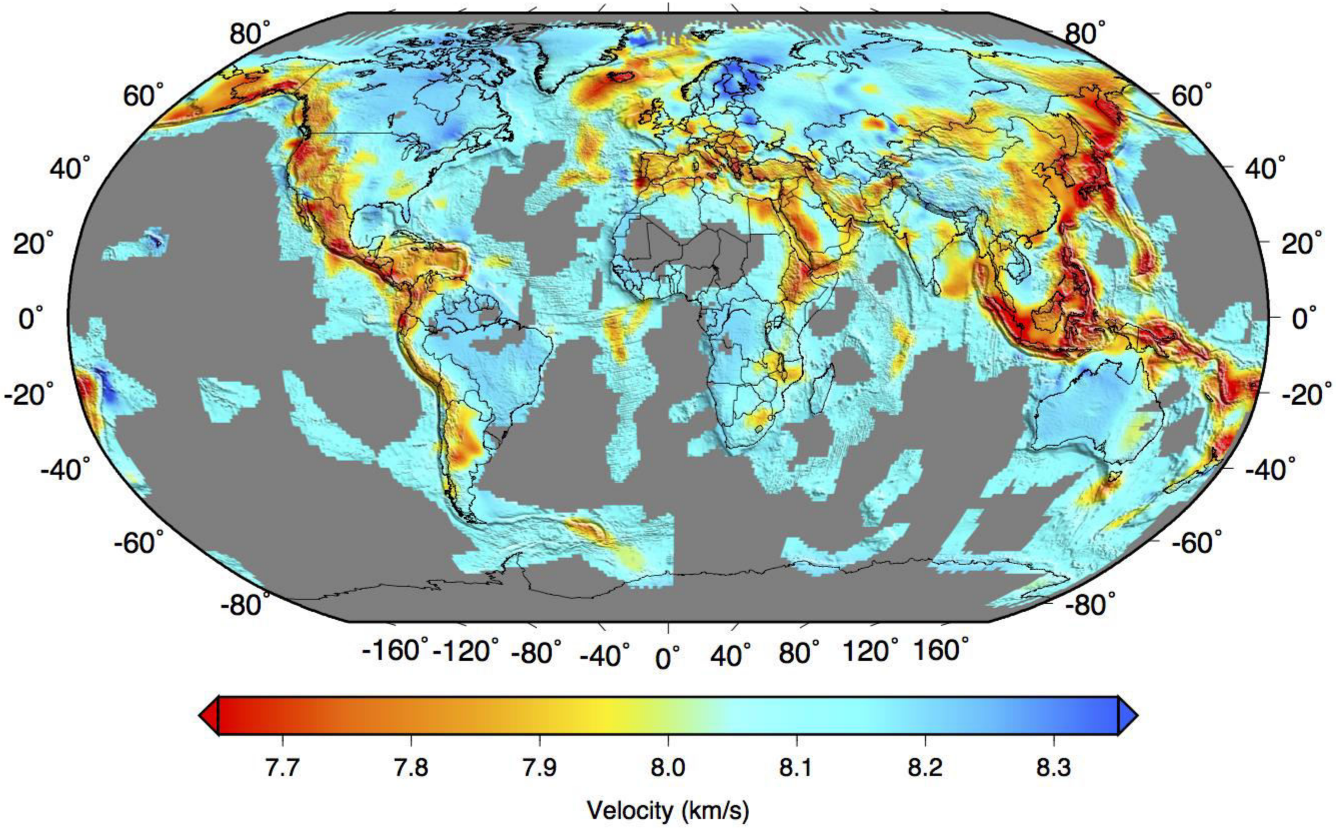
EXTRA SLIDES

Pn Upper Mantle Velocity



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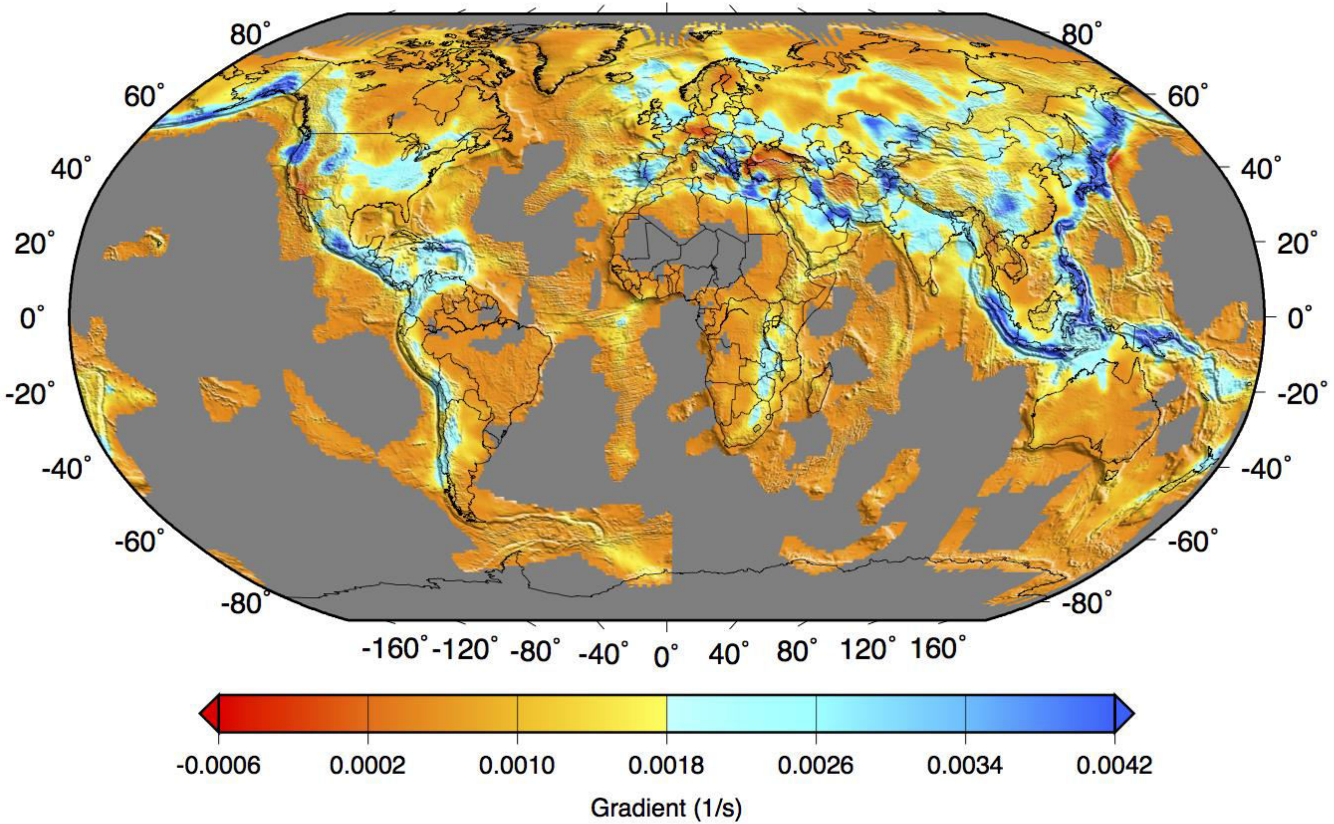
Pn Upper Mantle Gradient



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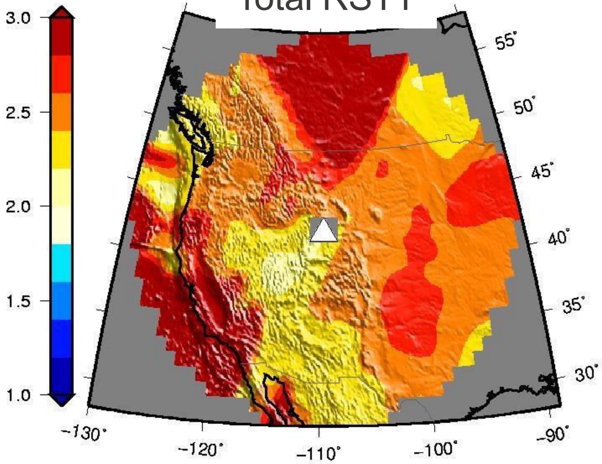
Proposed RSTT-SSSC Extrapolation (WGB-44, WEG; Le Bras et al.)

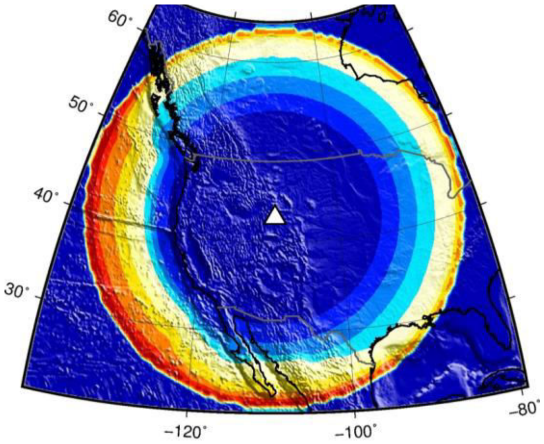
- **For DELTA < 15, use RSTT correction as provided (even if the library indicates that the estimate may be unreliable).**
- **For DELTA between 15 and 20 degrees:**
  - Compute dT/dDELTA (slowness) for the path and the path +/- 3 degrees azimuth.
  - Average the 3 slownesses using [0.5, 1.0, 0.5] weighting.
  - Compute travel time using: TT(15 deg) + dT/dDELTA(15 deg) * (delta - 15 deg)
  - Compute a trial SSSC using this travel time.
  - Use this correction if:
    - *The correction at 15 degrees has the same sign.*
    - *The absolute value of the correction is less than that at 15 degrees.*
  - If the correction changes sign from that at 15, then set correction to 0.
  - If the absolute value of the correction increases, then set it to the correction at 15 deg.
  - Since dT/dDELTA usually decreases with distance, then the time predicted from the above equation will overestimate the predicted travel time. Corrections to a 'fast path' (correction < 0) will tend to be reduced beyond 15 deg. Corrections to a slow path should tend to remain the same.
  - From DELTA > 18 linearly taper the correction toward 0 at 20 (delta2)

Pn Travel-time Uncertainty Surfaces (s): PDAR

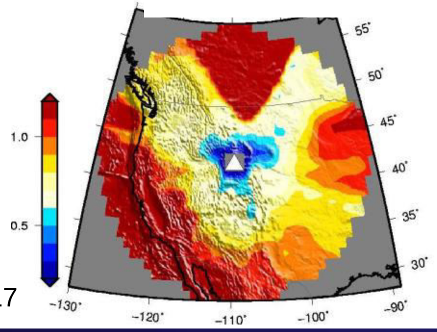
Total RSTT



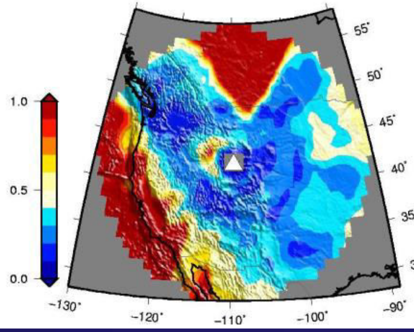
CTBTO SSSC



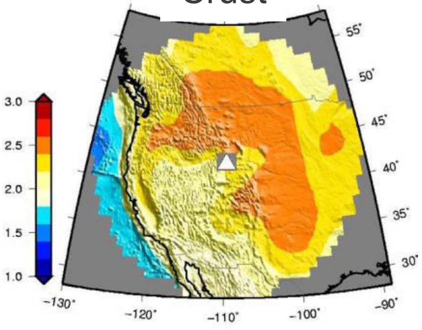
Variances



Bias



Crust



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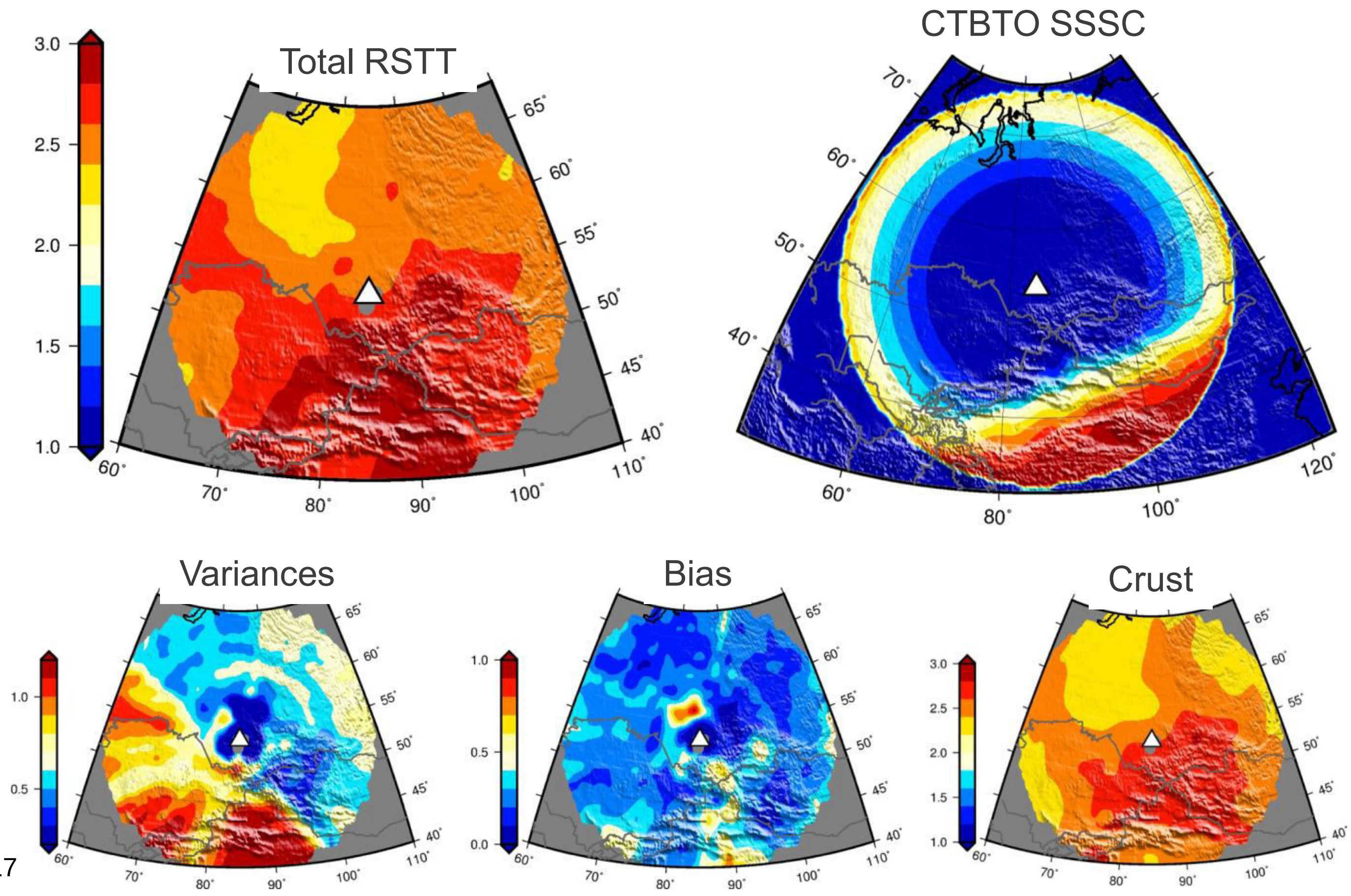
Pn Travel-time Uncertainty Surfaces (s): ZAL



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