SnT 2019
CTBT: SCIENCE AND TECHNOLOGY CONFERENCE

BGM - a new analysis method for beta-gamma radioxenon spectra, including improved calculation of decision limits

Anders Ringbom and Anders Axelsson
Swedish Defence Research Agency, FOI, Sweden

T3.5-P2

Along with the increasing experience from more than 15 years of radioxenon system operation and analysis, it has been found that there are reasons to re-visit the present analysis method (the NCC-method [1,2]) normally used to analyze radioxenon beta-gamma spectra. We present a new analysis method – the Beta-Gamma Matrix (BGM) method - with several benefits compared to the NCC-method:

1) An improved treatment of the calculation of decision limits.
2) Less complex equations thanks to a matrix formulation of the problem.
3) Fewer regions-of-interest (6 instead of 10).
4) Simplified introduction of new interference terms.
5) More straightforward treatment of covariances

The method is now used as the standard radioxenon beta-gamma analysis method at the Swedish National Data Center for CTBT monitoring.

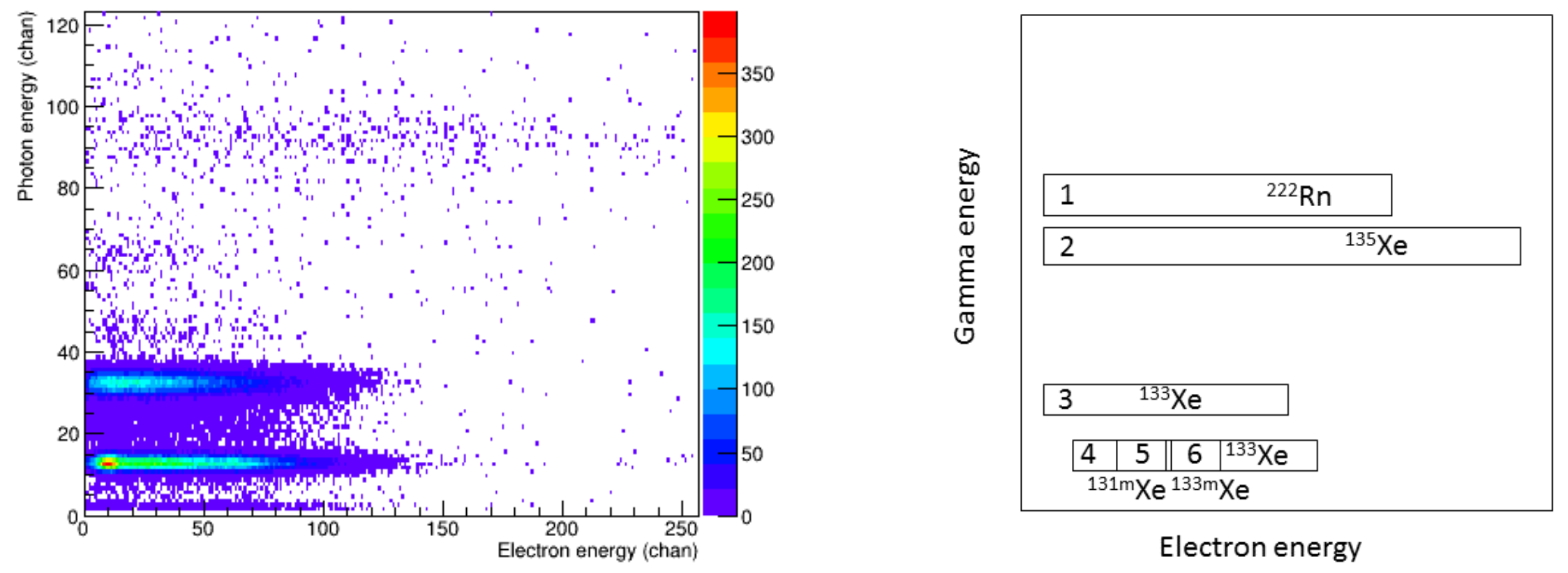

Fig. 1: A beta-gamma spectrum recorded with the SAUNA III radioxenon system (left), and definitions of the regions-of-interest used in the new analysis method presented here (right).

Basic equations

We define six regions-of-interest (ROIs, see fig. 1) .
The vector $\boldsymbol{y}$ of observed gross-counts in all ROIs, including gas-background if that is used, can be expressed as

$$\boldsymbol{y}=\boldsymbol{A}\boldsymbol{x}+\boldsymbol{k_d}\circ\boldsymbol{d}$$

where $\boldsymbol{x}$ is the ROI-specific activity, $\boldsymbol{d}$ is the detector background counts, $\boldsymbol{k_d}$ is the sample(gas background)/detector background counting time ratio*, and $\boldsymbol{A}$ is the *sensitivity matrix*, converting measured counts to activity. The activity is then estimated as

$$\hat{\boldsymbol{x}}=\boldsymbol{A}^{-1}(\boldsymbol{y}-\boldsymbol{k_d}\circ\boldsymbol{d})\overset{\text{def}}{=}\boldsymbol{B}\hat{\boldsymbol{z}}$$

The total error for $\hat{\boldsymbol{x}}$ is given by

$$(\Delta\hat{\boldsymbol{x}})^2=\underbrace{(\Delta\boldsymbol{B})^{\circ 2}\hat{\boldsymbol{z}}^{\circ 2}}_{\substack{\text{Uncertainty from efficiencies, branching ratios,}\\ \text{interference factors, and xenon volume.}\\ \Delta\boldsymbol{B}=-\boldsymbol{B}(\Delta\boldsymbol{A}^{-1}+\boldsymbol{B})^{-1}\boldsymbol{B}}}+\overbrace{\boldsymbol{B}^{\circ 2}(\boldsymbol{y}+\boldsymbol{k_d}^{\circ 2}\boldsymbol{d})}^{\text{Uncertainty from counting statistics}}$$

And the covariance matrix for $\boldsymbol{x}$ is calculated as

$$\sigma_{ij}=(\boldsymbol{b_i}^T\circ\boldsymbol{b_j}^T)(\boldsymbol{y}+\boldsymbol{k_d}^{\circ 2}\boldsymbol{d})$$

where $\boldsymbol{b_i}^T$ is the vector of row i in $\boldsymbol{B}$.

* The symbol ○ is the Hadamard product. $(\boldsymbol{A}\circ\boldsymbol{B})_{ij}=\boldsymbol{A}_{ij}\boldsymbol{B}_{ij}$

The part of the covariance matrix involving ROI 4, 5 and 6 is more complicated, since ROI 5 and 6 are contained inside ROI 4:

$$\sigma_{ij}=(\boldsymbol{b_i}^T\circ\boldsymbol{b_j}^T)(\boldsymbol{y}+\boldsymbol{k_d}^{\circ 2}\boldsymbol{d})$$
$$+(b_{i4}b_{j5}+b_{i5}b_{j4})(y_5+k_5^2d_5)+(b_{i4}b_{j6}+b_{i6}b_{j4})(y_6+k_6^2d_6)$$

Decision limits – initial approach

As a first approach we calculate the critical limit for an ROI i using the Poisson-Normal approximation according to [3]:

$$\hat{y}_{Ci}=k_\alpha\sqrt{\hat{y}_{0i}+var(\hat{y}_{0i})}$$

where the estimated background and its variance is given by

$$\hat{y}_{0i}=\boldsymbol{a}_{0i}^T\hat{\boldsymbol{x}}_i+k_i d_i$$
$$var(\hat{y}_{0i})=\boldsymbol{a}_{0i}^T\Sigma_{\hat{x}}\boldsymbol{a}_{0i}+k_i^2 d_i$$

where $\boldsymbol{a}_{0i}^T$ is row i in the sensitivity matrix, but with $a_{ii}=0$, and $\Sigma_{\hat{x}}$ is the covariance matrix for $\boldsymbol{x}$ calculated above. Monte-Carlo simulations show that this approach will result in too high false detection rates for low-background ROIs (se Fig. 2 and table 1), and sometimes gives an undefined answer. The reason is that the uncertainty of the background estimate is large, sometimes even resulting in a background less than zero.

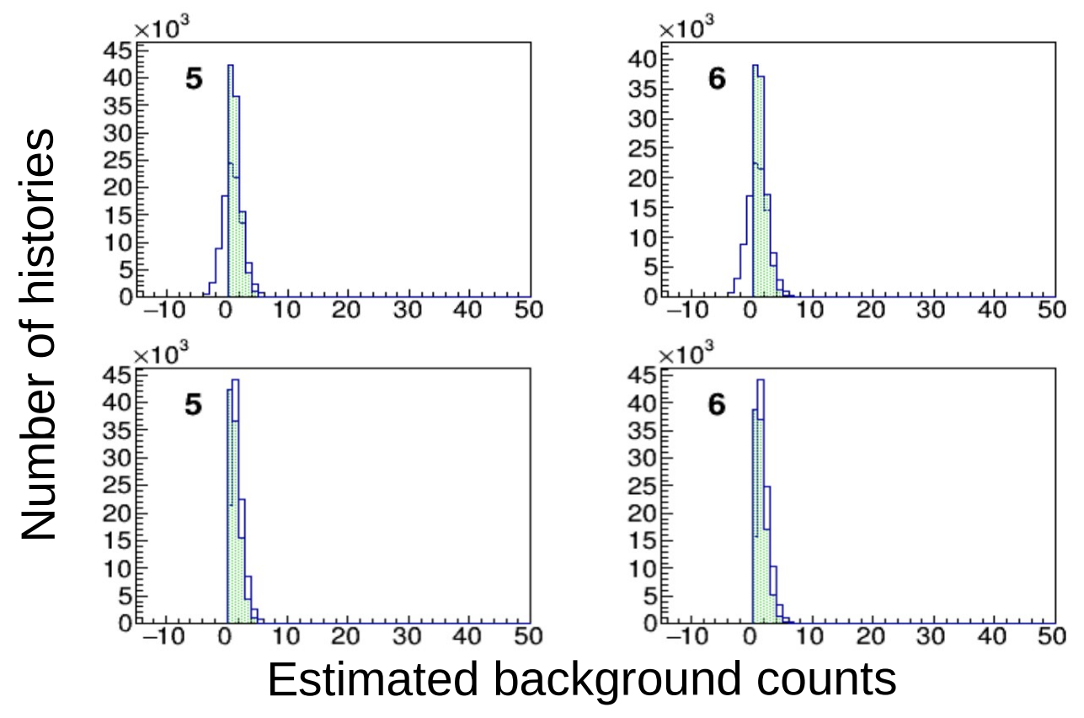

Fig. 2: Monte-Carlo simulations of true (green) and estimated distributions of the zero-signal background of ROI 5 (^{131m}Xe) and 6 (^{133m}Xe) in the new SAUNA III – system. Upper panels are results for the initial approach, and the lower panels are the result after the Bayesian correction has been applied.

ROI	SAUNA III Method A	Method B	SAUNA II Method A	Method B
1	6.2	6.3	6.5	6.4
2	5.8	6.0	5.7	5.6
3	7.0	6.7	6.7	6.6
4	8.0	6.2	6.6	4.9
5	11.2	7.0	8.8	6.5
6	11.3	6.6	9.7	7.7
7	5.0	5.0	5.1	4.9
8	5.7	5.5	5.6	5.6
9	6.2	6.2	6.2	6.1
10	6.7	4.7	7.3	4.5
11	10.0	7.3	9.0	7.7
12	10.2	7.3	8.8	8.1
3+4	7.5	7.6	6.1	6.1
9+10	4.7	4.9	5.7	4.8

Table 1: Monte-Carlo simulated false detection rates (%) with the BGM – method. Method A uses the original approach, and Method B includes the Bayesian correction. The assumed confidence level is 95%, which ideally should result in a 5% false detection rate.

Decision limits – modified approach

To correct for the sometimes nonphysical background estimate we introduce a Bayesian prior demanding $\hat{y}_{0i}\geqslant 0$, which results in a truncated Gaussian as the posterior probability density distribution [4]. The background estimate is then modified according to

$$\hat{y}_{0i}^*=\hat{y}_{0i}+\frac{\sqrt{var(\hat{y}_{0i})}}{\sqrt{2\pi}\Phi(\hat{y}_{0i}/\sqrt{var(\hat{y}_{0i})})}$$

This gives better agreement between estimated an true background (Fig. 2), and results in false detection rates closer to the assumed confidence level (table 1).

Results from measured data

Analysis of data from the new SAUNA III (see poster), show that the BGM results in similar activities as the NCC- method, but the critical limits are slightly larger for low-activity samples.

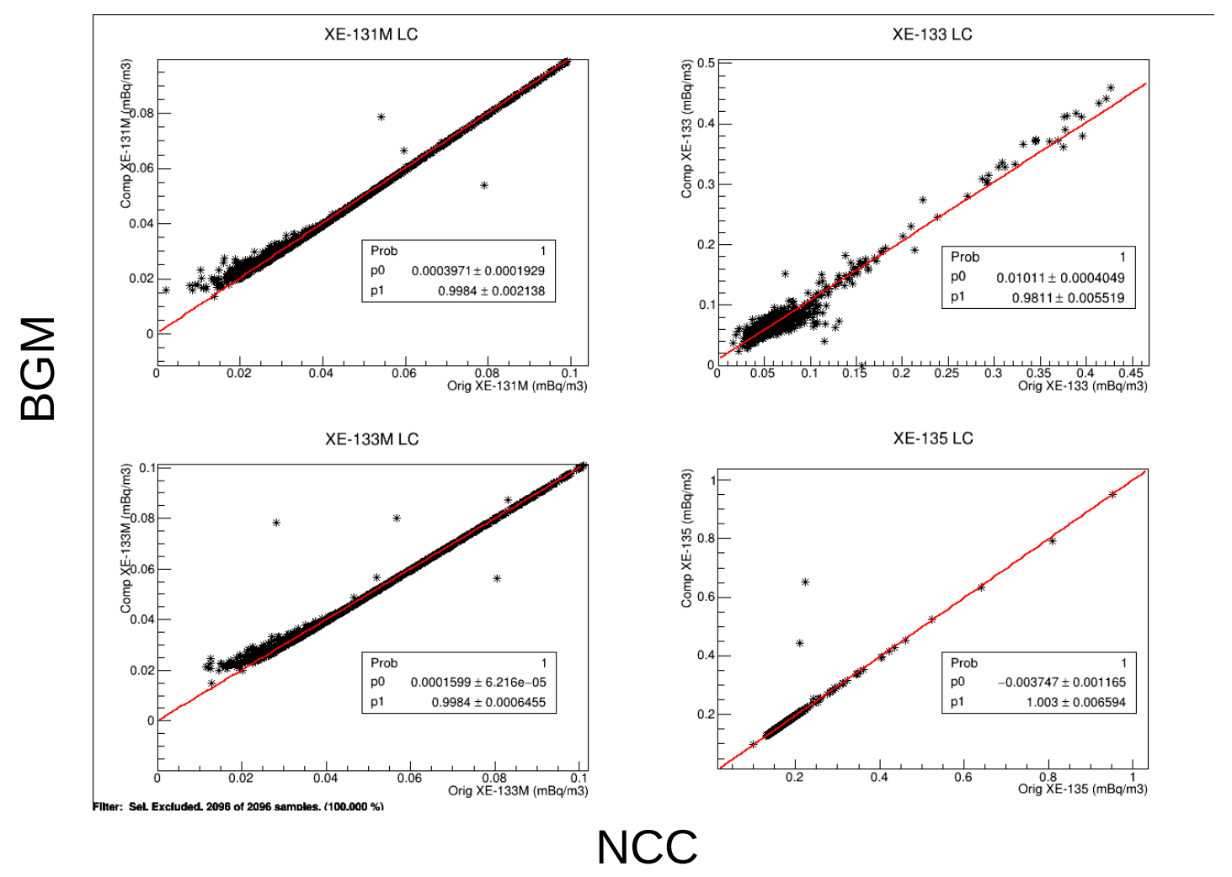

Fig. 3: A comparison of the critical limits obtained with the NCC- and the BGM – method, using data from the SAUNA III – system.

References

[1] Axelsson, A., Ringbom, A., September 2003. Xenon air activity concentration analysis from coincidence data. FOI-R--0913--SE.

[2] De Geer, L.-E., October 2007. The Xenon NCC method revisited. FOI-R--2350--SE.

[3] Currie, L. A., 1968. Limits for qualitative detection and quantitative determination. application to radiochemistry. Analytical Chemistry 40 (3), 586–593.
URL `https://doi.org/10.1021/ac60259a007`

[4] Zähringer, M., Kirchner, G., 2008. Nuclide ratios and source identification from high-resolution gamma-ray spectra with bayesian decision methods. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 594 (3), 400 – 406.
URL `http://www.sciencedirect.com/science/article/pii/S0168900208008760`