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A new blind deconvolution approach for the separation of seismic waves

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T3.5 - P4

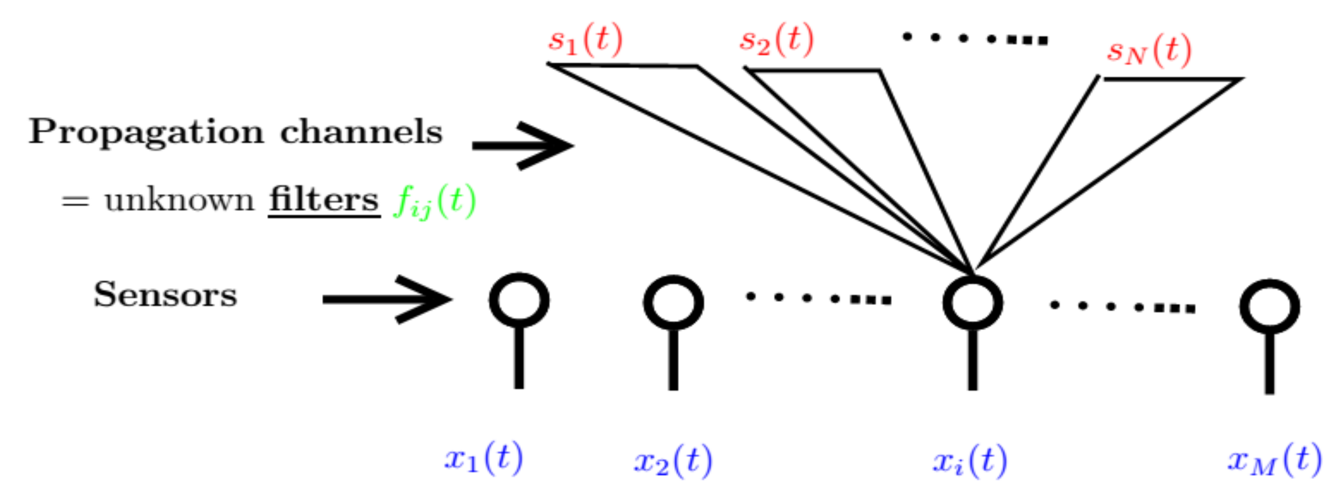
Abstract

In this paper we propose a new approach for the separation of different seismic waves (mainly P and S) which is based on the blind deconvolution of the signals provided by an array of seismic sensors. For this, we model the signal provided by each sensor by a noisy convolutive mixture of different seismic waves, where the noise signal, which we consider a source in its own right, is not necessarily stationary and white as in most works existing. In addition, unlike most of these recent works, which proposed for our mixing model (which is a model of a MIMO system) methods of blind separation of sources that require several assumptions about the sources and filters of mixing, we use a method of blind deconvolution based on subspace techniques for a SIMO (Single Input Multiple Output) system only, which requires a lot less assumptions and is more efficient. The results of our tests on artificial mixtures and some real mixtures of seismic signals are very encouraging.

1. Problematic

Seismic Waves = Sources

(unknown signals)



Mixtures = Observations

(known signals)

Goal : Estimate the N unknown sources $s_j(t)$ knowing only their M mixtures $x_i(t)$.
So it's an **inverse problem**

Problem : As the $f_{ij}(t)$ mixture filters are also unknown, it's a **ill-posed inverse problem** that admits an infinity of solutions and which is known as the problem of **Blind Source Separation** (**BSS**)

Solution : Impose **hypotheses** about sources $s_j(t)$ and / or **mix filters** $f_{ij}(t)$

2. Existing approaches

In seismic, most BSS approaches have modeled the mixtures $x_i(t)$ by **linear convolutive mixtures** as follows:

$$x_i(t) = \sum_{j=1}^{N}\sum_{k=0}^{K} f_{ij}(k)s_j(t-k), \quad i = 1, ..., M. \tag{1}$$

where K is the order of the longest filter supposed Finite Impulse Response (FIR). One of the possible solutions for blindly separating such a convolutive mixture consists in **reformulating** it as an **instantaneous mixture** as follows [1–3]:

$$x_i(t-d) = \sum_{j=1}^{N}\sum_{k=0}^{K} f_{ij}(k)s_j(t-(k+d)), \quad (i,d) \in [1,M] \times [0, L-1]. \tag{2}$$

$\Longrightarrow$ MD **generalized observations** $x_i(t-d), (i,d) \in [1,M] \times [0, D-1]$, and $N(K+D)$ **generalized sources** $s_j(t-r) = s_j(t-(k+d)), (j,r) \in [1,N] \times [0, K+D-1]$.

Defining $\mathbf{X}(t) = [\mathbf{x}_1^T(t), ..., \mathbf{x}_M^T(t)]^T$ and $\mathbf{S}(t) = [\mathbf{s}_1^T(t), ..., \mathbf{s}_N^T(t)]^T$ where

$$\mathbf{x}_i(t) = [x_i(t), x_i(t-1), ..., x_i(t-(D-1))]^T, \ \forall i \in [1, M], \tag{3}$$
$$\mathbf{s}_j(t) = [s_j(t), s_j(t-1), ..., s_j(t-(K+D-1))]^T, \ \forall j \in [1, N], \tag{4}$$

we obtain

$$\mathbf{X}(t) = \mathbf{F}_1 \cdot \mathbf{s}_1(t) + \mathbf{F}_2 \cdot \mathbf{s}_2(t) + ... + \mathbf{F}_N \cdot \mathbf{s}_N(t) = \mathbf{F} \cdot \mathbf{S}(t) \tag{5}$$

where

$$\mathbf{F} = (\mathbf{F}_1, \mathbf{F}_2, ..., \mathbf{F}_N) = \begin{pmatrix} \mathbf{F}_{11} & \dots & \mathbf{F}_{1N} \\ \vdots & \ddots & \vdots \\ \mathbf{F}_{M1} & \dots & \mathbf{F}_{MN} \end{pmatrix} \tag{6}$$

and $\mathbf{F}_{ij}$ is a matrix of dimension $D \times (K+D)$ defined by

$$\mathbf{F}_{ij} = \begin{pmatrix} f_{ij}(0) & \dots & f_{ij}(K) & 0 & \dots & 0 \\ & \ddots & & \ddots & & \\ & & \ddots & & \ddots & \\ 0 & \dots & 0 & f_{ij}(0) & \dots & f_{ij}(K) \end{pmatrix}. \tag{7}$$

$\mathbf{F}$ is a matrix of dimension $M' \times N'$ where $M' = MD$ and $N' = N(K+D)$. Thus, the mixture is (over-)determined if $M' \geq N'$, which implies $M > N$.

Objective: Estimate the pseudo-inverse of $\mathbf{F}$, denoted $\mathbf{F}^+$, in order to "ideally" find the source vector $\mathbf{S}(t)$ as follows:

$$\mathbf{F}^+ \cdot \mathbf{X}(t) = \mathbf{F}^+ \cdot \{\mathbf{F} \cdot \mathbf{S}(t)\} = \mathbf{S}(t). \tag{8}$$

Based on the **Independent Component Analysis** (**ICA**), these **BSS approaches** have as **Hypotheses**:

- **H1** : More sensors than seismic sources, i.e. $M > N$;
- **H2** : The sources $s_j(t)$ are **independent** : Non-rigorous hypothesis !
- **H3** : The mixing matrix **F** is of full rank : Non-realistic hypothesis !

As these hypotheses are difficult to verify, the **performances** of these **global approaches** of **BSS** are generally **very moderate**.

3. Proposed approach

3.1 Hypotheses

Based on the **Sparse Component Analysis** (**SCA**), our **local approach** has as **Hypotheses**:

- **H1** : Have *two* sensors at least , i.e. $M \geqslant 2$;
- **H2** : The **sources** $s_j(t)$ are **Sparse**, i.e :

$$\forall j, \ \exists \text{ a zone } \mathbf{Z}_j \ / \ \forall t \in \mathbf{Z}_j, \ \ \mathbf{X}(t)|_{t \in \mathbf{Z}_j} = \mathbf{X}_j(t) = \mathbf{F}_j \cdot \mathbf{s}_j(t), \tag{9}$$

$$\text{i.e. } \forall i \neq j, \ \ \forall t \in \mathbf{Z}_j, \ \ \mathbf{s}_i(t) = 0$$

- **H3** : The mixing submatrix $\mathbf{F}_j$ is of full rank $\Rightarrow \mathbf{F}_j^+ \cdot \mathbf{X}_j(t) = \mathbf{F}_j^+ \cdot \{\mathbf{F}_j \cdot \mathbf{s}_j(t)\} = \mathbf{s}_j(t)$

Thus, our **blind deconvolution approach** is based on a **SIMO model** on each zone $\mathbf{Z}_j$. His **hypotheses** are much **more realistic** compared to those of **existing approaches**.

3.2 Principle and Algorithm

To simplify, we consider the case of two mixtures $x_i(t)$ of three sources $\mathbf{s}_i(t)$ (i.e. $M = 2$ and $N = 3$). In seismic, we can for example identify these three sources respectively to the **Noise signal**, the **P wave** and the **S wave**. We therefore have:

$$\mathbf{X}(t) = \mathbf{F}_1 \cdot \mathbf{s}_1(t) + \mathbf{F}_2 \cdot \mathbf{s}_2(t) + \mathbf{F}_3 \cdot \mathbf{s}_3(t) \tag{10}$$

1. The first zone $\mathbf{Z}_1$ is the zone of silence where only the noise signal is present (i.e in the absence of seismic event). In this area we have the following SIMO model:

$$\mathbf{X}_1(t) = \mathbf{F}_1 \cdot \mathbf{s}_1(t) \tag{11}$$

2. The identification of the submatrix $\mathbf{F}_1$ is then done using the **subspace method** [4].
3. By multiplying on the left the equation (10) by $\mathbf{F}_1^+$ we get:

$$\mathbf{F}_1^+ \cdot \mathbf{X}(t) = \mathbf{F}_1^+ \cdot \mathbf{F}_1 \cdot \mathbf{s}_1(t) + \mathbf{F}_1^+ \cdot \mathbf{F}_2 \cdot \mathbf{s}_2(t) + \mathbf{F}_1^+ \cdot \mathbf{F}_3 \cdot \mathbf{s}_3(t) \tag{12}$$
$$= \mathbf{s}_1(t) + \mathbf{H}_2 \cdot \mathbf{s}_2(t) + \mathbf{H}_3 \cdot \mathbf{s}_3(t) = \mathbf{Y}(t) \tag{13}$$

4. Thanks to the properties of the components of the vector $\mathbf{s}_1(t)$, we can **eliminate its contributions** from the vector $\mathbf{Y}(t)$. We thus obtain the following new vector $\mathbf{V}(t)$:

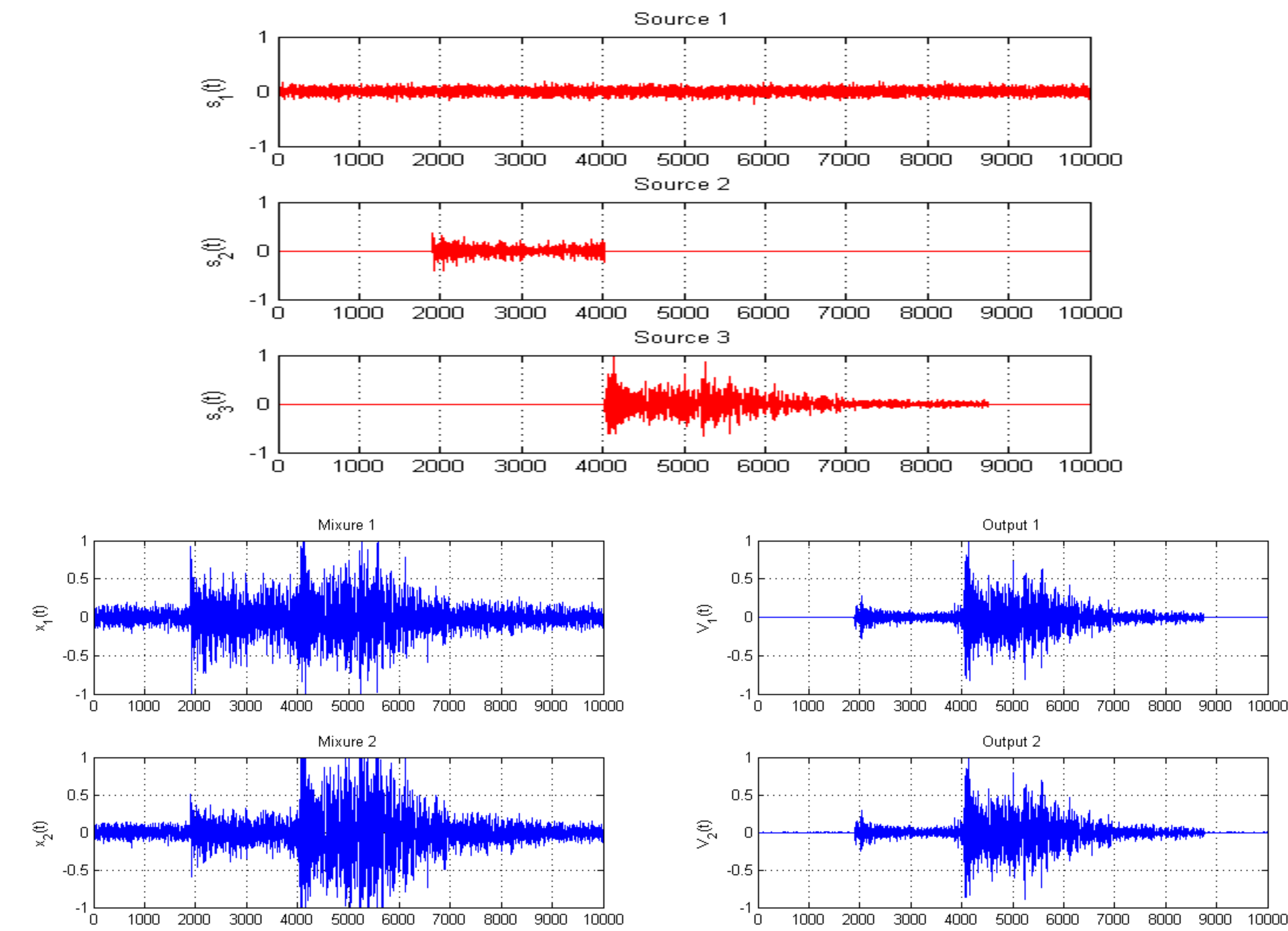
$$\mathbf{V}(t) = \mathbf{G}_2 \cdot \mathbf{s}_2(t) + \mathbf{G}_3 \cdot \mathbf{s}_3(t) \tag{14}$$

Equation (14) is similar to Equation (10) with **one less source**. Assuming now that there is a zone $\mathbf{Z}_2$ where **only the source** $\mathbf{s}_2(t)$ **is active**, and proceeding in the same manner as above, we will finally obtain a vector $\mathbf{W}(t)$ which contains **only the source** $\mathbf{s}_3(t)$, ie :

$$\mathbf{W}(t) = \mathbf{M}_3 \cdot \mathbf{s}_3(t) \tag{15}$$

4. Results

We first consider **two artificial mixtures** of **three** sources (**Noise signal**, **P wave** and **S wave**). Each mixture was generated using the relation (1) with a **filter order** K equal to 63. As shown in figure 3, the output signals $V_1(t)$ and $V_2(t)$ no longer contain noise (which is in accordance with equation (14)).



Applying our approach in a second time in the zone which contains only the P wave we will be able to eliminate it to finally get the S wave only, and vice versa.

5. Conclusion and Perspectives

- With much more **realistic hypotheses**, this **local approach** of **blind deconvolution** is **more efficient** than the classical and **global approaches** of **Blind Source Separation** which are based on **ICA**.
- Very **encouraging results**, **however**, **other tests** are needed for both **artificial mixtures** and **real mixtures** of seismic signals to validate these results.

References

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