Matrix operations of the net count calculation method

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Introduction

- The 2D beta/gamma coincidence spectra are analyzed using the so called net count calculation (NCC) method based on a number of Regions of Interest (ROI), which quantify the presence of CTBT relevant radioxenon isotopes (^{131m}Xe, ^{133m}Xe, ^{133}Xe and ^{135}Xe) in the noble gas samples.
- The number of gross counts is a linear sum of the net counts from the isotope associated to the ROI, the detector background, interference contributions from other radionuclides, and the activity remaining in the detector cell if applicable. The NCC equations are presented in a different way that eases their resolution by a matrix approach. In this poster we present the procedures of calculating net counts and their uncertainties, including decision thresholds.

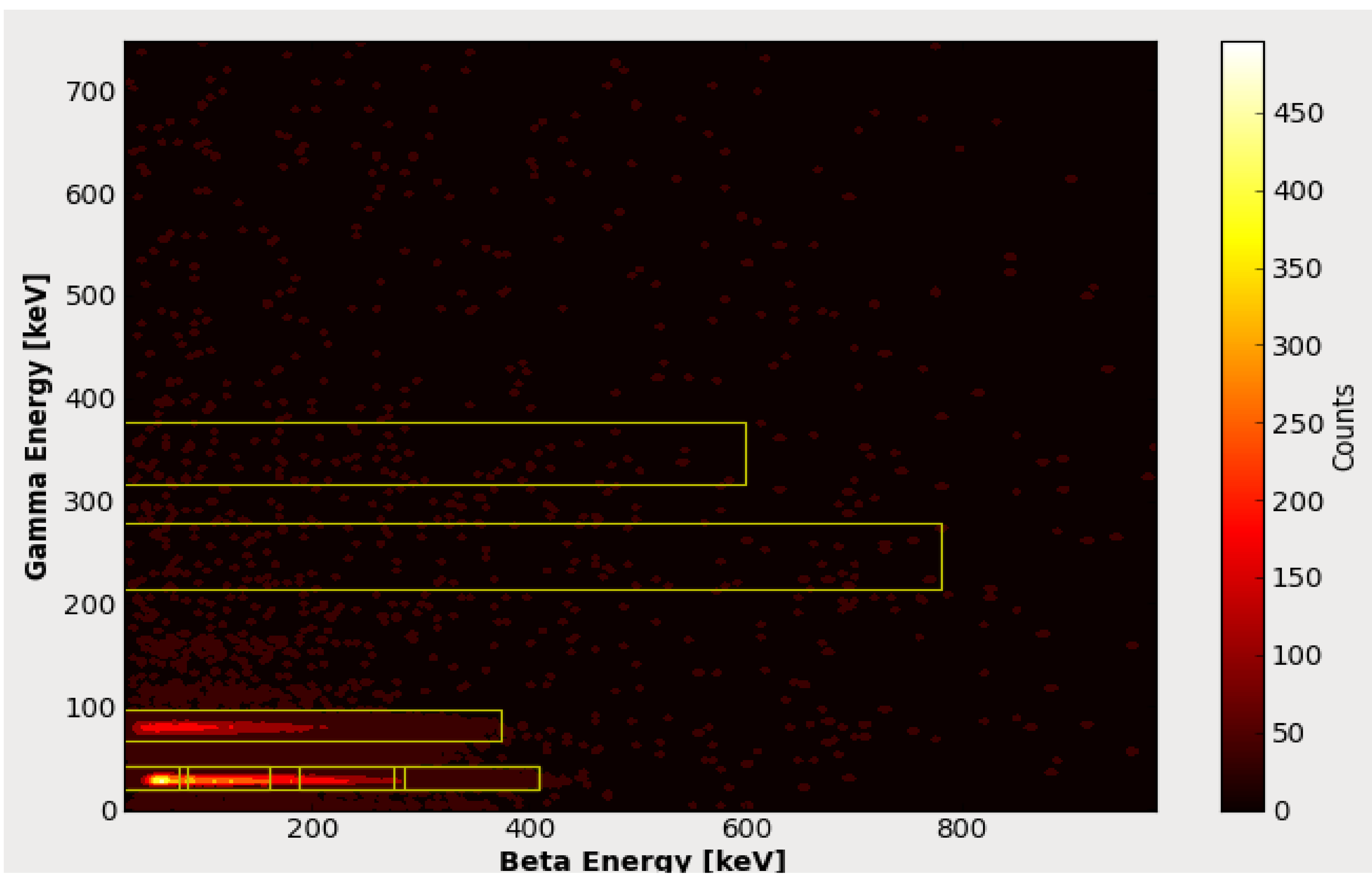

Fig. 1. *Beta/gamma coincidence spectrum of a spike with Xe-131m and Xe-133 from JPX38_004. The rectangular ROIs from top to bottom are ROI-1 to ROI-4, and ROI-4 covers ROI-5 to ROI-10. ROI-5 and 6 are the second and third rectangles (from the left hand) in ROI-4.*

Table 1. *Matrix ROIs (MR-1 to MR-6) and their typical ranges of beta and gamma energy for SAUNA (see Fig. 1) and SPALAX NG samples.*

Matrix ROI	SAUNA samples				SPALAX NG samples			
	ROI	Isotope	β keV	γ keV	ROI	Isotope	β keV	γ keV
MR-1	ROI-1	^{214}Pb	17-600	316-377	ROI-0	^{214}Bi	50-900	607.5-611.5
MR-2	ROI-2	^{135}Xe	17-782	213-279	ROI-1	^{214}Pb	50-900	350.5-353.5
MR-3	ROI-3	^{133}Xe	17-374	66-96	ROI-2	^{135}Xe	50-900	248.5-251.0
MR-4	ROI-6	^{133m}Xe	188-276	20-42	ROI-3	^{133}Xe	50-350	80.2-81.8
MR-5	ROI-5	^{131m}Xe	85-160	20-42	ROI-6	^{133m}Xe	180-210	28.0-30.3
MR-6					ROI-5	^{131m}Xe	110-140	28.0-30.3

Matrix Approach for the sample spectrum

- Numbers of gross counts
  $C_i = x_i + \sum_{j=1}^{i-1} R_{ij}x_j + B_{C_i}$
  - C_i is the number of gross counts in ROI-*i* in the spectrum of a sample,
  - B_{C_i} is the contribution of the detector background to C_i,
  - x_i is the number of net counts of the isotope in the associated ROI-*i*,
  - R_{ij} is the interference ratio of isotope *j* (associated with ROI-*j*) to ROI-*i*

- Numbers and their variances of net counts
  $x_i = C_i - B_{C_i} + \sum_{j=1}^{i-1}(R^{-1})_{ij}(C_j - B_{C_j})$
  $v_{x_i} = C_i + B_{C_i} + \sum_{j=1}^{i-1}(R^{-1})^2_{ij}(C_j + B_{C_j}) + \sum_{j=1}^{i-1} v_{(R^{-1})_{ij}}(C_j - B_{C_j})^2$
  - $(R^{-1})_{ij}$ is the element of the inverse matrix;
  - v_{x_i} is the variance of the net counts.
- Decision threshold estimation
  $$v_{x_i}(0) = 2B_{C_i} + \sum_{j=1}^{i-1}(R^{-1})_{ij}(C_j - B_{C_j}) + \sum_{j=1}^{i-1}(R^{-1})^2_{ij}(C_j + B_{C_j}) + \sum_{j=1}^{i-1} v_{(R^{-1})_{ij}}(C_j - B_{C_j})^2$$
  - $x_i^* = ku_{x_i}(0) = k\sqrt{v_{x_i}(0)}$

- Matrix operations
  $\boldsymbol{X} = \boldsymbol{R}^{-1}\cdot(\boldsymbol{C} - \boldsymbol{B_C})$; $\boldsymbol{V_{CB}} = [\boldsymbol{C} + \boldsymbol{B_C}] * \boldsymbol{I}$; $\boldsymbol{V_x} = \boldsymbol{R}^{-1}\cdot\boldsymbol{V_{CB}}\cdot[\boldsymbol{R}^{-1}]^{\boldsymbol{T}}$
  - $\boldsymbol{I}$ is an identity matrix;
  - Uncertainties of interference ratios ($v_{(R^{-1})_{ij}}$) are not included.
- Decision threshold estimation
  - Set $x_i = 0$ but the net counts for other isotopes are kept unchanged:

  $\boldsymbol{X} \xRightarrow{x_i=0} \boldsymbol{X}_i(0)$;
  $\boldsymbol{V_{CB_i}}(0) = [\boldsymbol{R}\cdot\boldsymbol{X}_i(0) + 2\boldsymbol{B_C}] * \boldsymbol{I}$; $\boldsymbol{V_{x_i}}(\boldsymbol{0}) = \boldsymbol{R}^{-1}\cdot\boldsymbol{V_{CB_i}}(0)\cdot[\boldsymbol{R}^{-1}]^{\boldsymbol{T}}$;

Implementing the non-negative net counts

- Temporary background in each ROI including interference
  $TB_i = \sum_{j=1}^{i-1} R_{ij}x_j + B_{C_i}$
- Non-negative Interference corrections
  $x_i = Max(0, (C_i - TB_i))$
  - Non-negative net counts in the higher energy ROI.
  - Example of SAUNA samples:

  $x_1 = Max(0, (C_1 - f_sB_1))$; $x_3 = Max(0, (C_3 - (f_sB_3 + R_{31}x_1)))$
  - f_s is the normalization factor of the detector background measurement.
- Preparation for the matrix approach
  - Set the minimum numbers of gross counts as their temporary background.
  - Perform the matrix calculation by using the updated gross counts.

Xe-133m decay correction

- Xe-133 Activity at acquisition start:
  $a_{133_S} = f_{a(\varepsilon_{133})}(x_{133} - f_{133gm_{APLC}}x_{133m}) = f_{a(\varepsilon_{133})}x_{133_S}$;
  - $f_{133gm_{APLC}}$ is the decay correction factor during processing and acquisition.
- Decision threshold:
  $u^2_{x_{133_S}}(0) = u^2_{x_{133}}(0) + f_{133gm_{APLC}}x_{133m} + f^2_{133gm_{APLC}}u^2_{x_{133m}}$

Subtraction of the gas background spectrum

- The gas background spectrum is processed separately
  $\boldsymbol{X_G} = \boldsymbol{R}^{-1}\cdot(\boldsymbol{D} - \boldsymbol{B_D})$; $\boldsymbol{V_{DB}} = [\boldsymbol{D} + \boldsymbol{B_D}] * \boldsymbol{I}$; $\boldsymbol{V_{x_G}} = \boldsymbol{R}^{-1}\cdot\boldsymbol{V_{DB}}\cdot[\boldsymbol{R}^{-1}]^{\boldsymbol{T}}$
  - D_i is the gross counts of the gas background measurement.
  - B_{D_i} is the contribution of the detector background to D_i,
  - x_{G_i} is the number of net counts of the gas background measurement.
- Subtraction of the memory effect remaining in the measurement cell
  - Non-negative: $x_{G_i}(G) = Max(0, x_{G_i})$
  - Net counts: $x_i(G) = x_i - F_ix_{G_i}(G)$
  - Variance: $v_{x_i}(G) = v_{x_i} + F_i^2v_{x_{G_i}}(G)$
  - Decision threshold: $v_{x_i}(0)(G) = v_{x_i}(0) + F_ix_{G_i}(G) + F_i^2v_{x_{G_i}}(G)$
  - F_i is the decay correction factor between the sample and gas background measurements.

Activity calculation

- Activity at acquisition start
  $a_j = f_{a(\varepsilon_j)}x_j; f_{a(\varepsilon_j)} = \frac{1}{\varepsilon_j\cdot BR_j}\frac{\lambda_j}{1-e^{-\lambda_j\cdot T_A}}\frac{T_A}{T_{A_l}}$; $u^2_{a_j} = f^2_{a(\varepsilon_j)}u^2_{x_j} + a_j^2\left(\frac{u_{\varepsilon_j}}{\varepsilon_j}\right)^2$
- Activity concentration during sampling (average)
  $ac_j = f_{ac_j}a_j$; $f_{ac_j} = \frac{1}{V_{air}}\left(\frac{\lambda_j\cdot T_C}{1-e^{-\lambda_j\cdot T_C}}\right)\left(\frac{1}{e^{-\lambda_j\cdot T_P}}\right)$;$u^2_{ac_j} = f^2_{ac_j}u^2_{a_j} + ac_j^2\left(\frac{u_{V_{air}}}{V_{air}}\right)^2$
- Characteristic limits
  $a_j^* = kf_{a(\varepsilon_j)}u_{x_j}(0)$
  $MDA_j = f_{a(\varepsilon_j)}(k^2 + 2ku_{x_j}(0))/(1 - k^2(\frac{u_{\varepsilon_j}}{\varepsilon_j})^2)$
  $MDC_j = f_{ac(j)}f_{a(\varepsilon_j)}(k^2 + 2ku_{x_j}(0))/(1 - k^2(\left(\frac{u_{\varepsilon_j}}{\varepsilon_j}\right)^2 + \left(\frac{u_{V_{air}}}{V_{air}}\right)^2))$

Summary

- It is concise to calculate the numbers of net counts and their uncertainties by using the matrix approach. The ROI net counts are then converted to radioxenon activities at acquisition start. Activity concentrations are finally calculated under the assumption that activity concentrations of each nuclide are constant in the air during sampling.
- Calculating the uncertainties of interference ratios by using the matrix operations needs to be investigated further.
- Best estimates and confidence intervals regarding the low count samples will be investigated in the future.

References

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