



INTRODUCTION

This work is the result of a collaboration between the Department of Statistical Sciences of the University of Bologna and the National Data Centre - Radionuclides (NDC-RN) of ENEA in Bologna and is part of a research project carried out by NDC-RN and the International Data Centre of CTBTO. The purposes of this work are I) to perform a detailed statistical analysis of the radionuclide atmospheric background and of the radionuclide abnormal activity concentrations, measured at six IMS stations and II) to compare the IDC method (based on an "Inter-Quartile Filter") with the "Statistical Process Control" approach (based on "Control Charts"), in order to detect radionuclide abnormal activity concentration values (anomalous values). The comparison is based on the percentage of anomalous observations detected. The data from IMS station JPX38 in Takasaki, Japan, have been shown as an example of the applied methodology. The download of the restricted data has been performed by means a "vDEC" contract formalized between the University of Bologna and the CTBTO.

METHODS AND DESCRIPTION

- I. A statistical descriptive analysis has been performed for the JPX38 station considering the history from the certification date 19/12/2014 to 25/07/2018 (Figure 1), in order to characterize:
 - a. The radionuclide (Xe-133) atmospheric background
 - b. The radionuclide (Xe-133) anomalous activity concentrations taking into account the "Abnormal Limit" (IDC method) calculated on 25 July 2018. The data distribution is highly positively skewed (Figure 2).
- II. In order to apply the "Statistical Process Control" method, a "Box-Cox" power transformation has been applied to reduce the asymmetry (Figure 3).
- III. In order to characterize the Xe-133 anomalous values, two type of "Control Charts" have been applied with the aim to differentiate two sources of variability: common causes (i.e natural background) and assignable causes (i.e events not compatible with the natural background).
 - c. The "Shewhart Control Chart", relatively insensitive to small process shifts, has been applied with the aim to assess the natural radionuclide background (Figure 4).
 - d. The "EWMA Control Chart", effective for detecting small variations in the background noise, has been applied with the aim to assess the anomalous values (Figure 5). Accordingly to the IDC method based on an inter-quartile filter (IDC-IQF), the "Shewhart Control Chart" has been set up with 3-sigma limit and the "EWMA Control Chart" has been set up to have the same performance of the Shewhart control chart. In such a way, the IDC-IQF method and the "Statistical Process Control" approach are coherent but non directly comparable because the first one is a non-parametric method while the second one is a parametric approach.

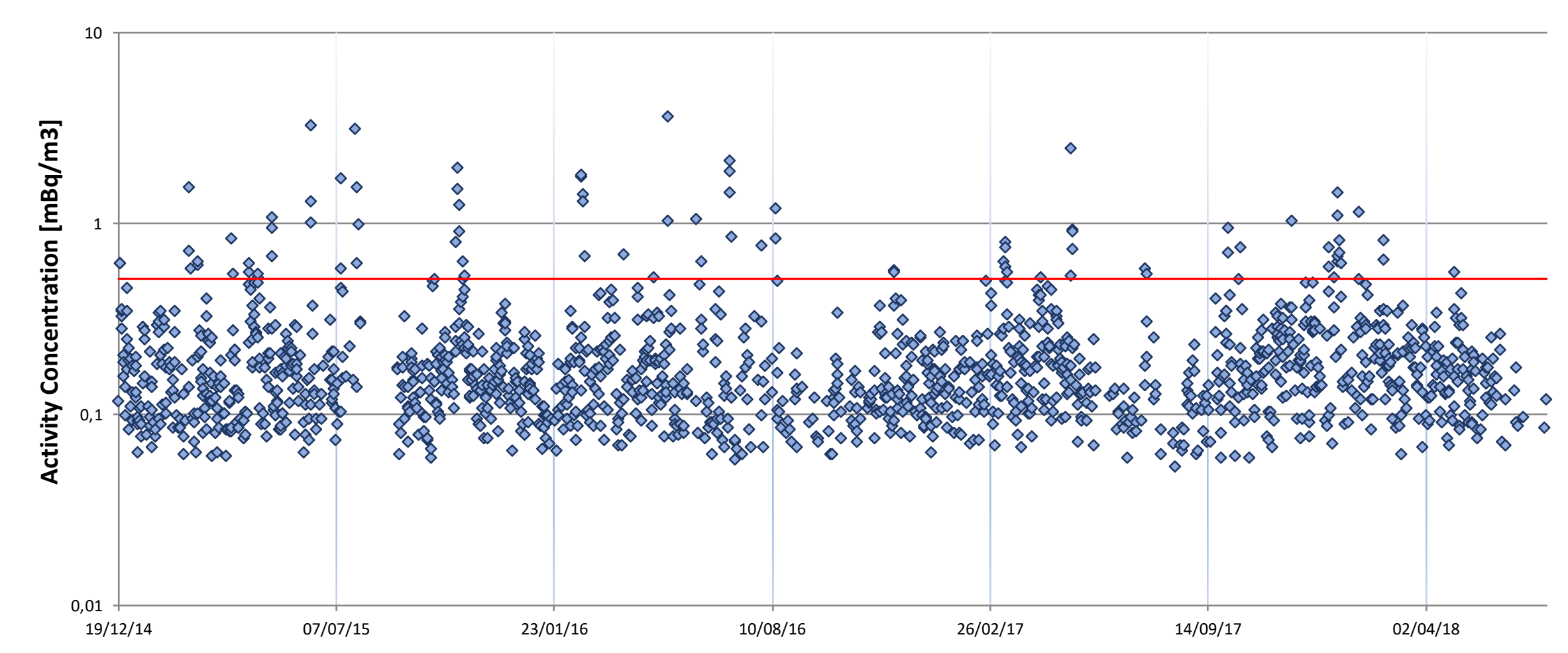


Figure 1: Time history of JPX38; on x-axis, the time, from the certification date 12/19/14 to 25 July 2018; on y-axis, the radionuclide activity concentrations. The red line is the abnormal limit calculated on 25 July 2018.

The "Inter-Quartile Filter" methods used by the IDC to assess the abnormal concentration value, or "Abnormal Limit", is defined as follows:

$$L_A = Q_2 + \lambda_A [Q_3 - Q_1]$$

where:

- Q₁ is the first quartile
- Q₂ is the median
- Q₃ is the third quartile
- λ_A = 3

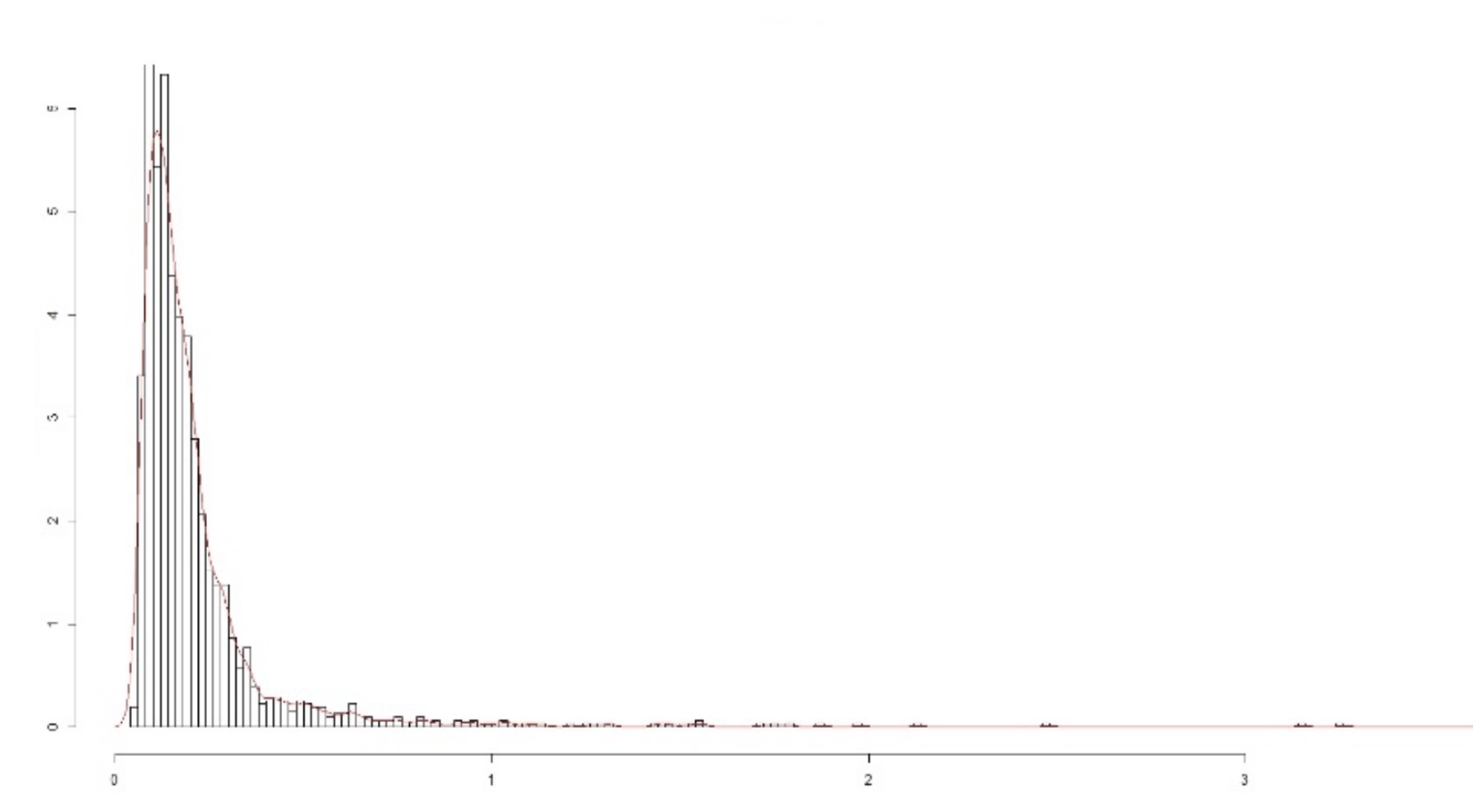


Figure 2: Density histogram of radionuclide activity concentrations detected at JPX38.

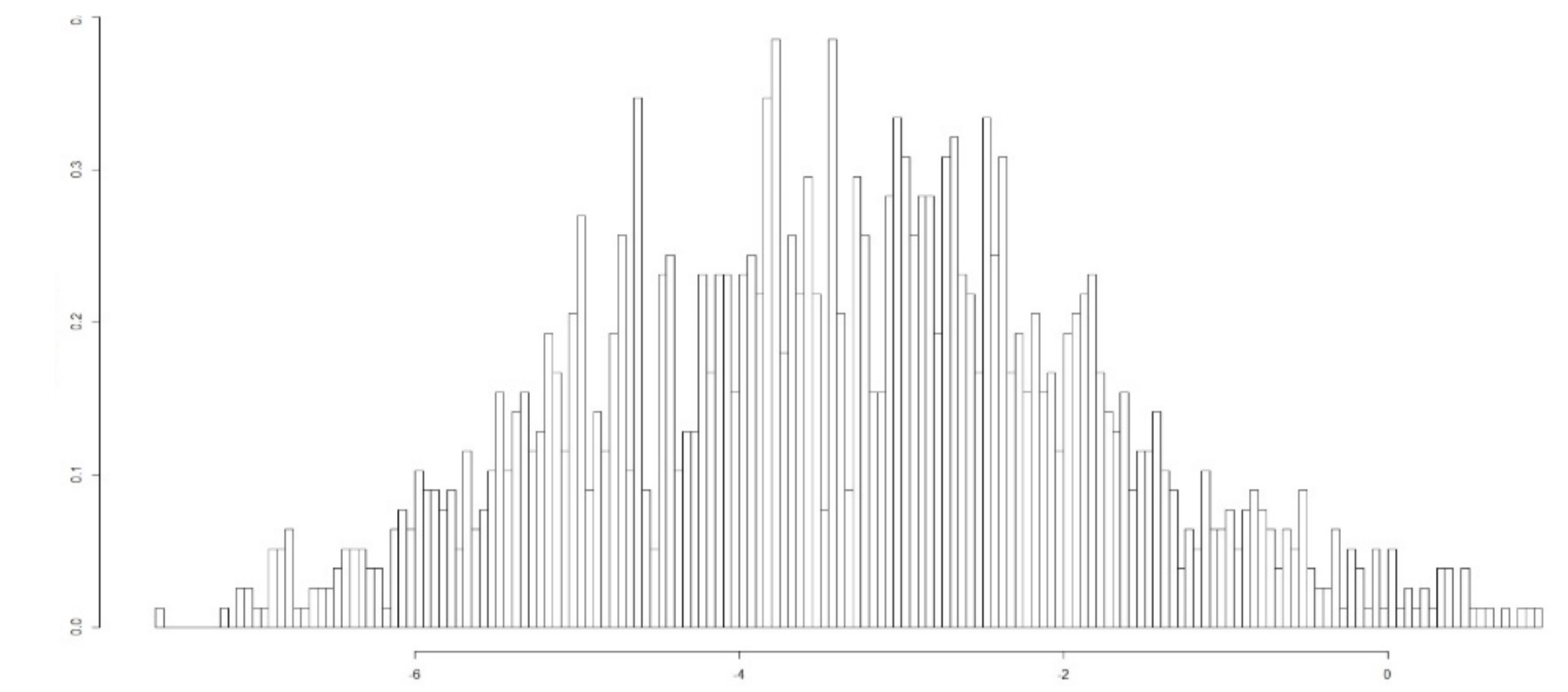


Figure 3: Density histogram of the transformed values of the radionuclide activity concentrations detected at JPX38.

SHEWHART

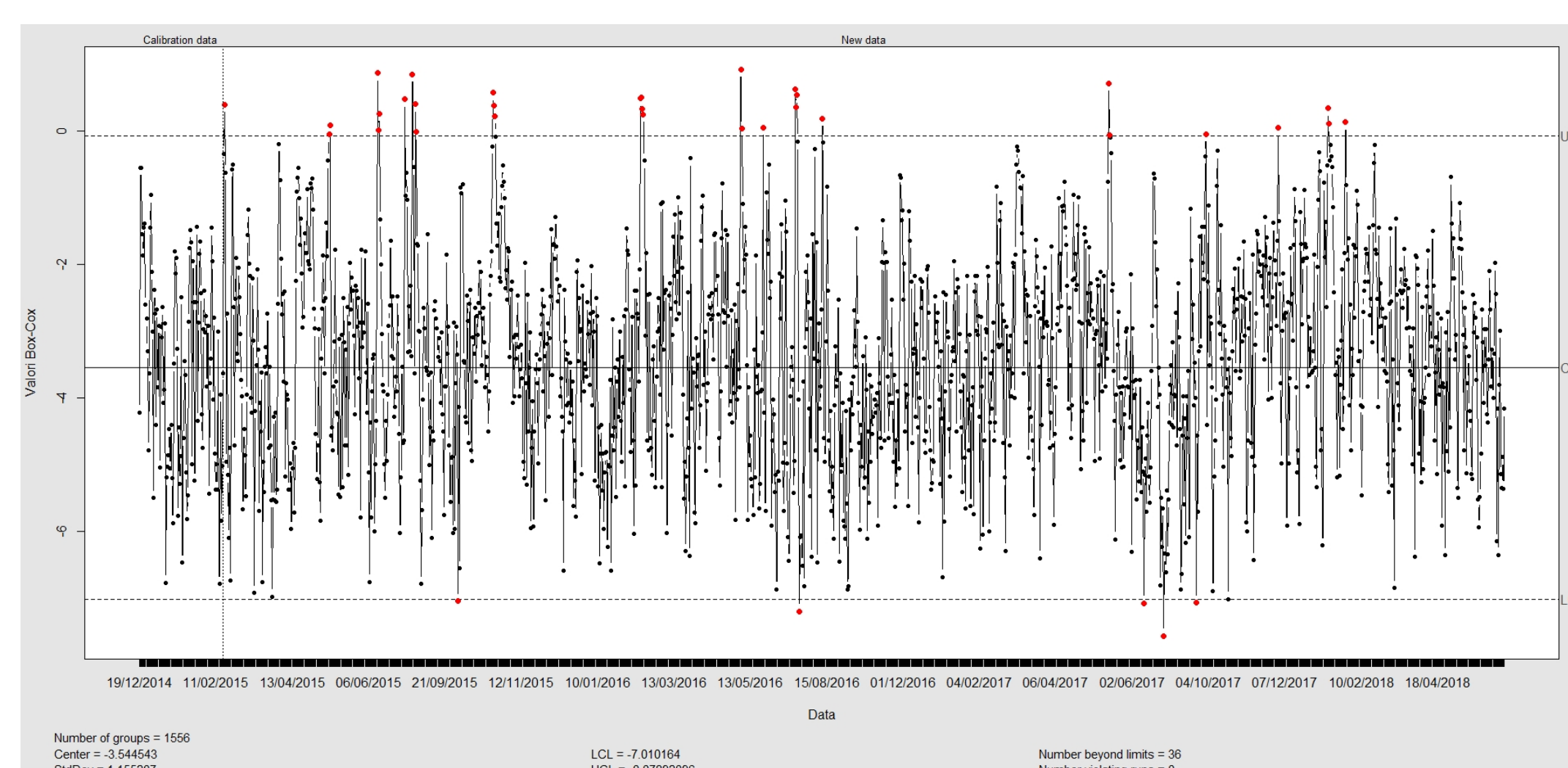


Figure 4: Shewhart control chart for individual measurements - Phase II

The main quantities used for the Shewhart chart are:

$$CL = \bar{X} = \sum_{i=1}^m \frac{x_i}{m}$$

$$UCL = \bar{X} + 3 \frac{MR}{d_2}$$

$$LCL = \bar{X} - 3 \frac{MR}{d_2}$$

where

$$\overline{MR} = \sum_{i=1}^{m-1} \frac{MR_i}{n-1}$$

$$MR_i = |x_i - x_{i-1}|$$

$$d_2 = 1,128$$

This chart shows 36 values above the «Upper Control Limit».

EWMA

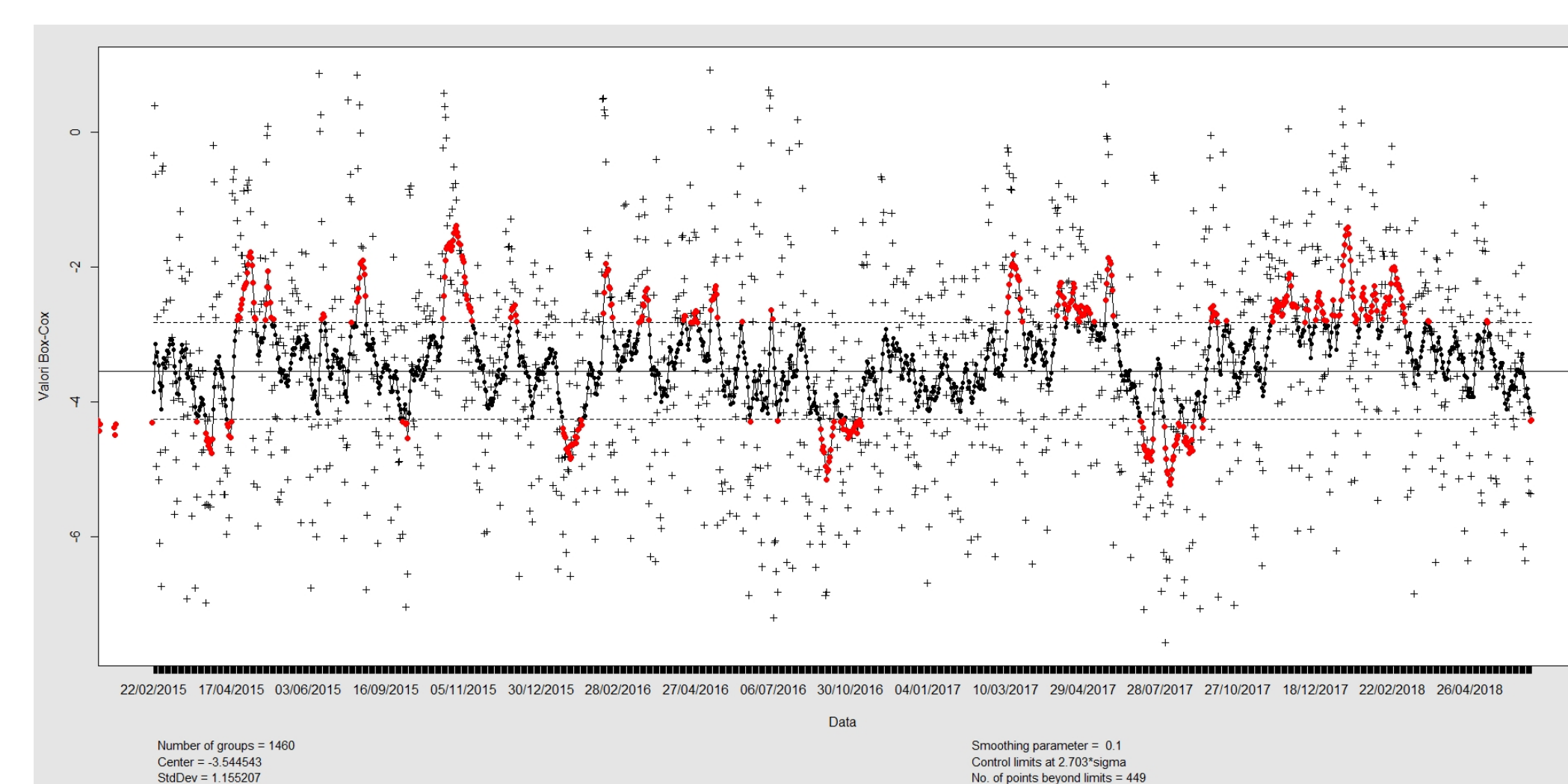


Figure 5: EWMA control chart for individual measurements - Phase II

The EWMA statistics, Z_i, for the i-th observations is defined recursively

$$Z_i = \lambda x_i + (1 - \lambda) Z_{i-1}$$

where x_i is the i-th observation, λ is a weight parameter (0 < λ < 1). The main quantities used to built the EWMA chart are:

$$CL = \bar{X}$$

$$UCL = \bar{X} + L \hat{\sigma} \sqrt{\frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2i}]}$$

$$LCL = \bar{X} - L \hat{\sigma} \sqrt{\frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2i}]}$$

In order to obtain an EWMA chart with the same "in-control" performance of the Shewhart chart: let be λ = 0,1 and L=2,703 and therefore ARL₀=370,4.

where L is the constant for the control limits. The EWMA chart shows 304 values above the «Upper Control Limit».

RESULTS

The percentage ratio of the anomalous values to the total number of available observations, has been calculated using the two methods (IDC-IQF and Control Charts) and then compared (Figure 6).

The results are the following:

- I. The Shewhart control chart shows a number of out-of-control observations (above the upper limit) less than the anomalous observations highlighted by the IDC-IQF method;
- II. The EWMA control chart shows more out-of-control observations (above the upper limit), than the IDC-IQF and Shewhart.

"Inter-Quartile Filter" vs "Shewhart's Chart" vs "EWMA Chart"

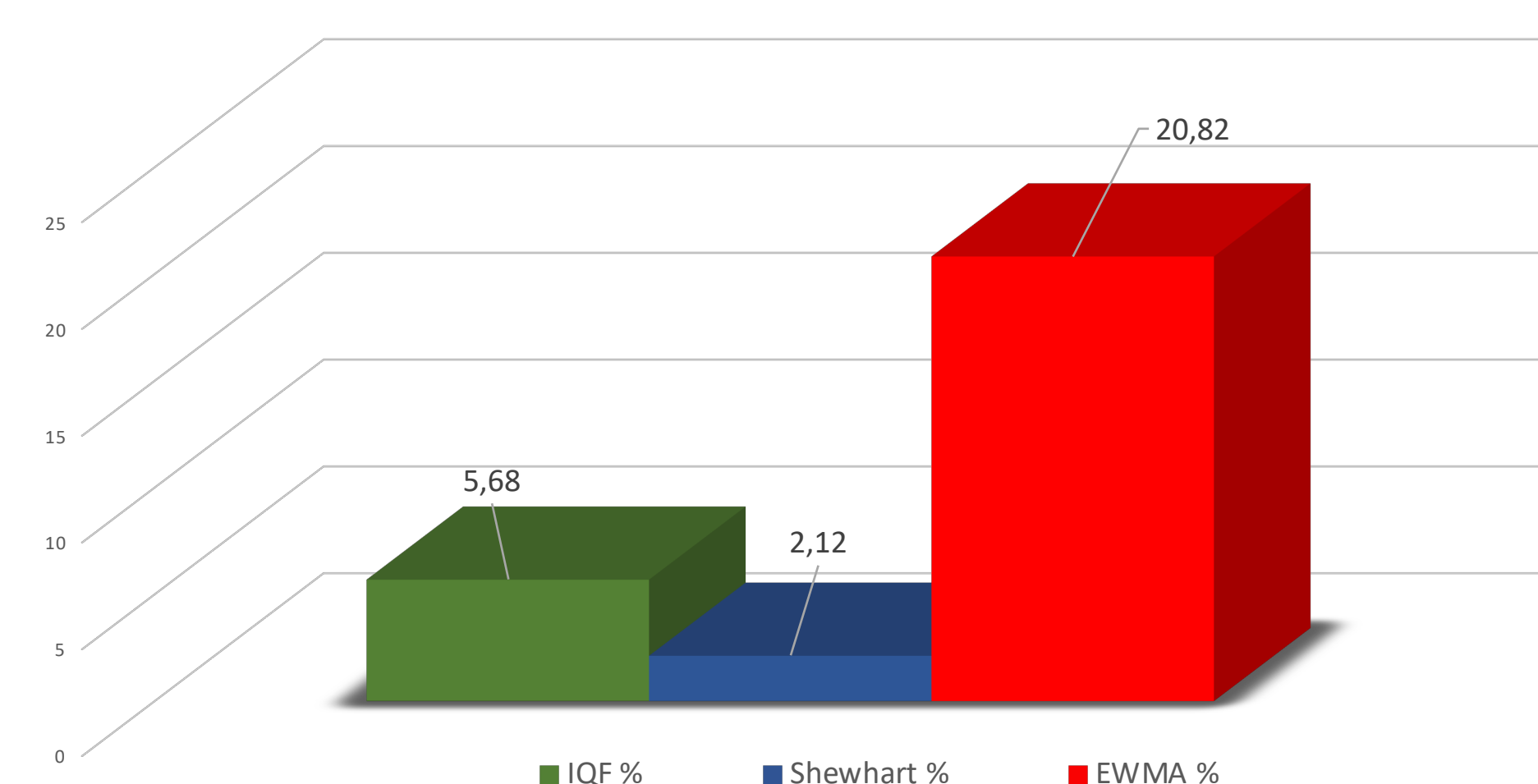


Figure 6: Comparison of the analysis results for the JPX38 station, using the IDC methodology (green), Shewhart chart (blue) and EWMA chart (red).

The analysis performed to understand which observations are highlighted or missed with each of these methods, is currently deepening.

Furthermore, it can be suggested that

- A. the Shewhart control chart is suitable to detect relatively large shift from the background and therefore it can be useful for the periodic monitoring of IMS stations in order to identify large variations of radio-xenon activity concentrations;
- B. the EWMA control chart can detect very gradual changes therefore it can be more suitable for specific studies on the atmospheric background and on the anomalies of radio-xenon activity concentrations.

The use of the "Control Charts" method as a possible complement to the IDC "Inter-Quartile Filter" method, needs to be further investigated.