Seismic Spectral Ratios between North Korean Nuclear Tests: Observations and Implications

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1. Spectral Ratios of NKT2017 to other Korean Nuclear Tests

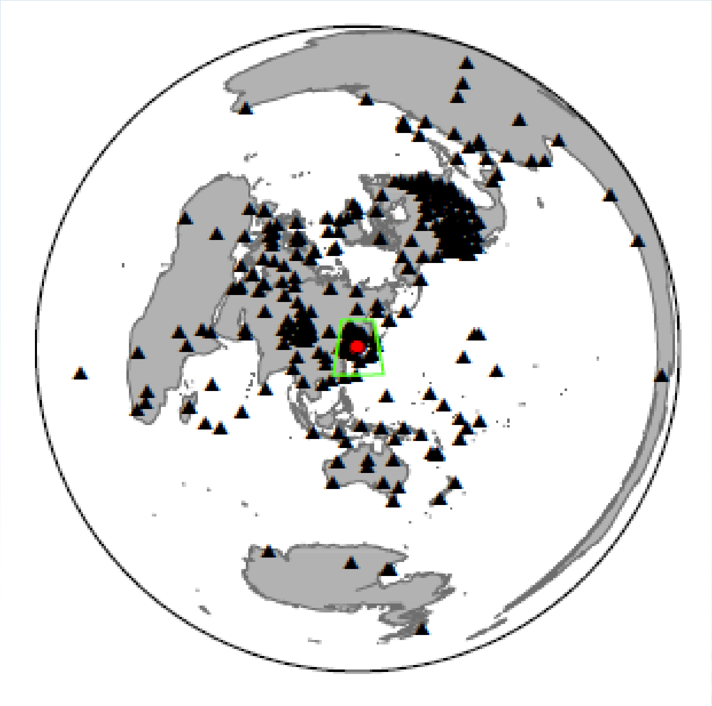
- Spectral ratios between a pair of nearby events were used widely for investigations for seismic sources, taking the advantage that complicated path effects can be cancelled out while information about the sources are retained.
- The North Korean nuclear test on September 3, 2017 (NKT2017) provides a unique opportunity to investigate Korean nuclear tests using seismic spectral ratios.

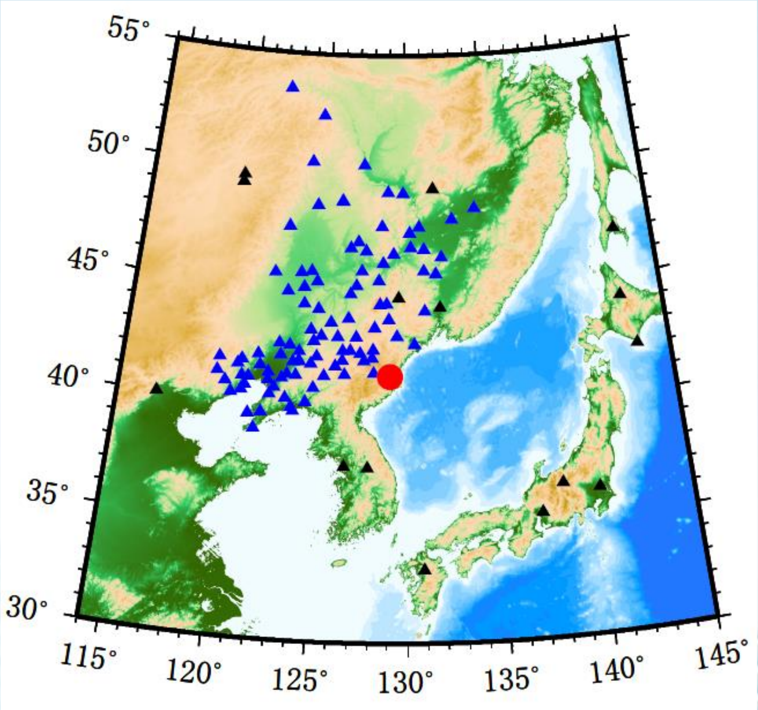
✓ magnitude of NKT2017 much larger than earlier Korean nuclear tests;

✓ for NKT2017 and the four Korean nuclear tests between 2009 and September 2016 (NKLT2009,NKT2013,NKT2016J and NKT2016S), high quality waveform data are available at many regional and teleseismic stations;

✓ the separation between the explosions is less than 1 km according to accurate relative location results;

- Data used for this study.





teleseismic regional

Figure 1. Regional (left) and global (right) seismic stations used in this study. Data from theses stations can be accessible via IDC, IRIS and the Institute of Geophysics, China Earthquake Administration (www.cea-igp.ac.cn).

- P wave spectral ratios of NKT2017 to NKT2009~ NKT2016S.

✓ Obvious differences observed between regional and teleseismic spectral ratios;

✓ The teleseismic observations exhibit a unique notch at about 2.5 Hz, which manifest the pP-wave effect of NKT2017;

✓ The pP-wave effect doesn't manifest itself on regional observations as the 2.5Hz notch is absent.

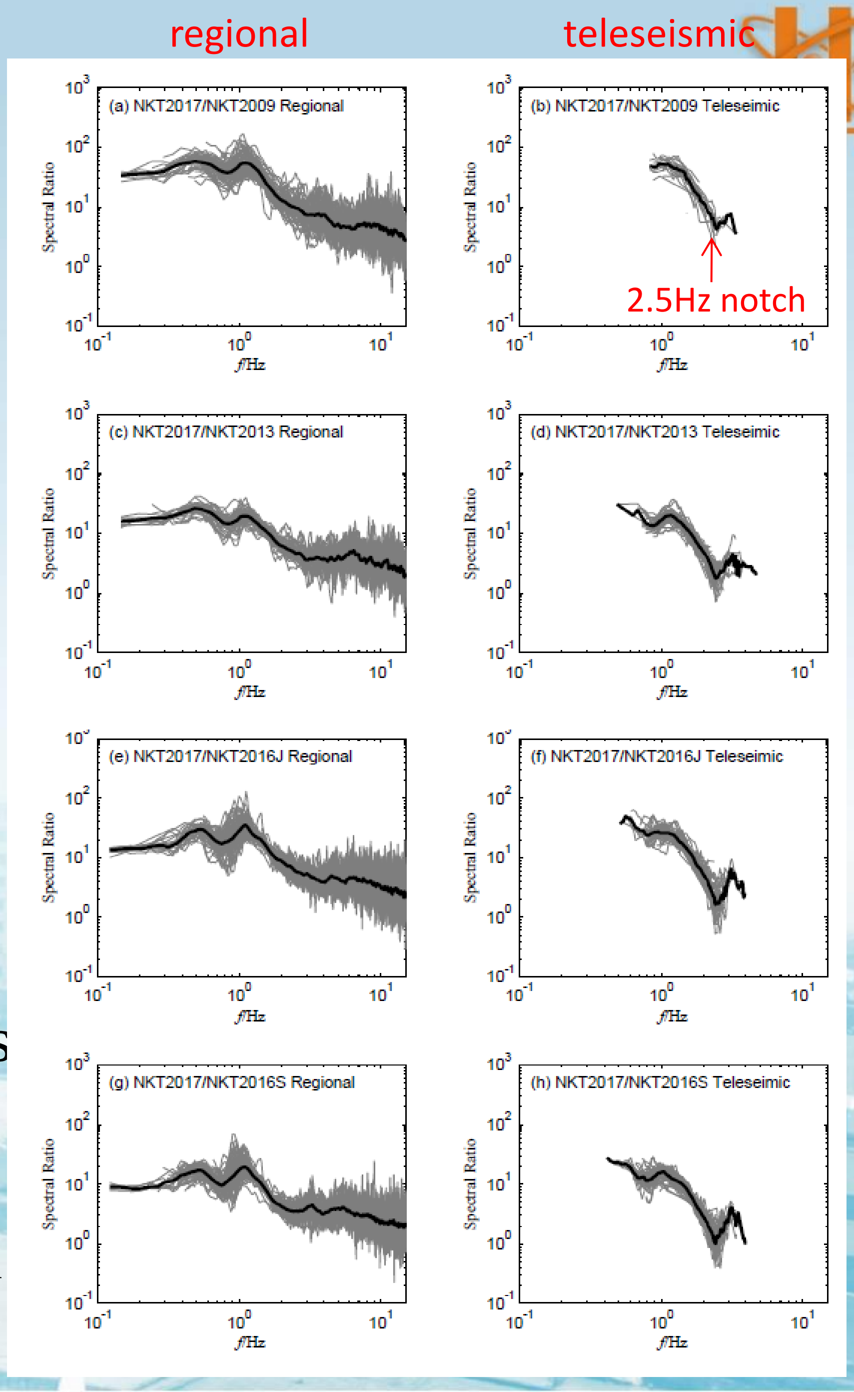


Fig. 2 Observed P wave spectral ratios between Korean nuclear tests.

- Observed similarity between P- and Lg-wave spectral ratios.

✓ The network-averaged Lg-wave spectral ratio is similar to that of regional P-wave in both shape and amplitude, but with the source corner frequencies reduced significantly.

✓ The observed similarity is consistent with the Fisk conjecture (Fisk, 2006).

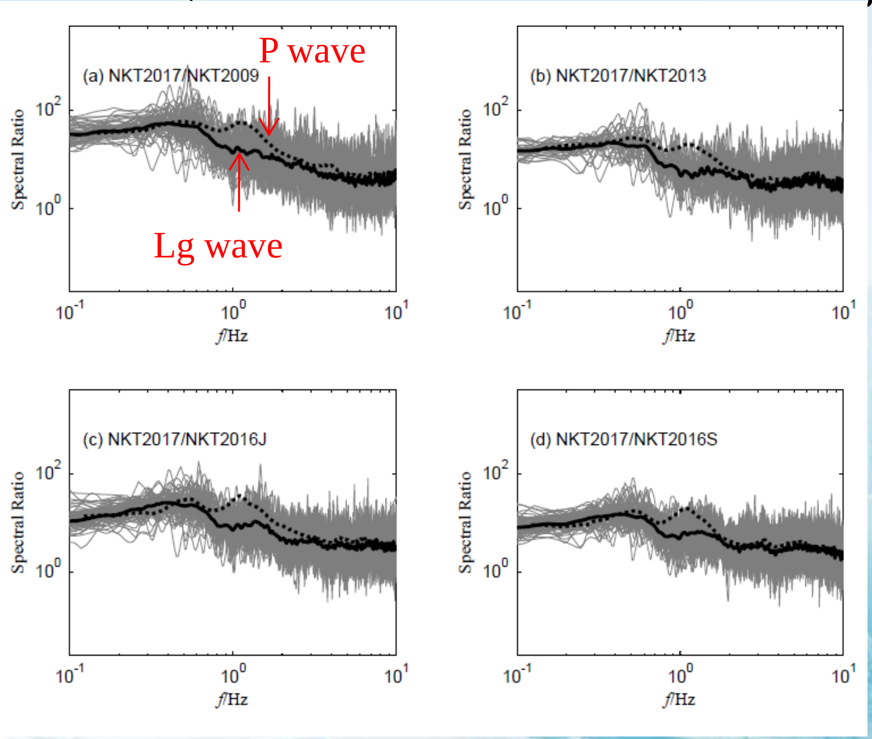
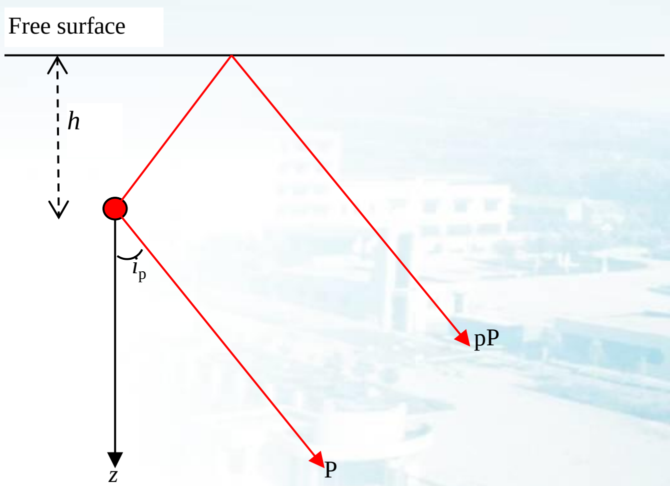


Figure 3.Observed Lg spectral ratios between Korean nuclear tests.

2. The Classical Theory about Spectral Ratios between Underground Explosions.

- As a first-order approximation, teleseismic P wave of underground explosions can be viewed as the sum of a downgoing P wave and an apparent pP wave;
- Under this approximation, the spectral ratio between a pair of nearby underground explosions is the product of the source spectral ratio and the ratio related to pP reflection;



$$\frac{U_i(\omega)}{U_j(\omega)} = \frac{S_i(\omega)}{S_j(\omega)} \times \frac{(1 + r_{pP}^{(i)} e^{-i\omega \Delta t_{pP}^{(i)}})}{(1 + r_{pP}^{(j)} e^{-i\omega \Delta t_{pP}^{(j)}})}$$

$S_i(\omega)$ – source spectrum of E_i;

$r_{pP}^{(i)}(\omega)$ – reflection coefficient of apparent pP wave ;

$\Delta t_{pP}^{(i)}(\omega)$ – apparent pP-P time delay;

Fig. 4 Illustration of P wave of an underground explosion.

- If the reflection of pP wave is elastic (e.g. the explosion is heavily overburied), the pP-P time difference will be simply related to the DOB of an underground explosion;

$$\Delta t_{pP}^{(i)} = t_{pP}^{(i)} - t_{P}{}^{(i)} = 2h_i \cos i_P / \bar{\alpha}_s$$

- However, for common underground explosions, the reflection of pP wave involves complicated nonlinear process, the apparent DOB estimated from pP-P time difference would be much larger than the true DOB.

$$h_{apparent} = \bar{\alpha}_s \Delta t_{pP} / 2\cos i_P$$

- In addition, the pP-wave reflection coefficient would be significantly less than that by elastic theory, and would be frequency dependent;

- # Source models of underground nuclear explosions.

✓ MM71 model (Mueller & Murphy, 1971);

✓ DJ91 model (Denny & Johnson, 1991);

✓ MM71+DJ91a (Hybrid model I, obtained by inserting RDP of DJ91 into MM71 model, Ford & Walter, 2013);

✓ MM71+DJ91b(Hybrid model II, obtained by inserting cavity size of DJ91 into MM71, Patton & Rougier, 2015);

(a) Source spectra (b) Source spectral ratios

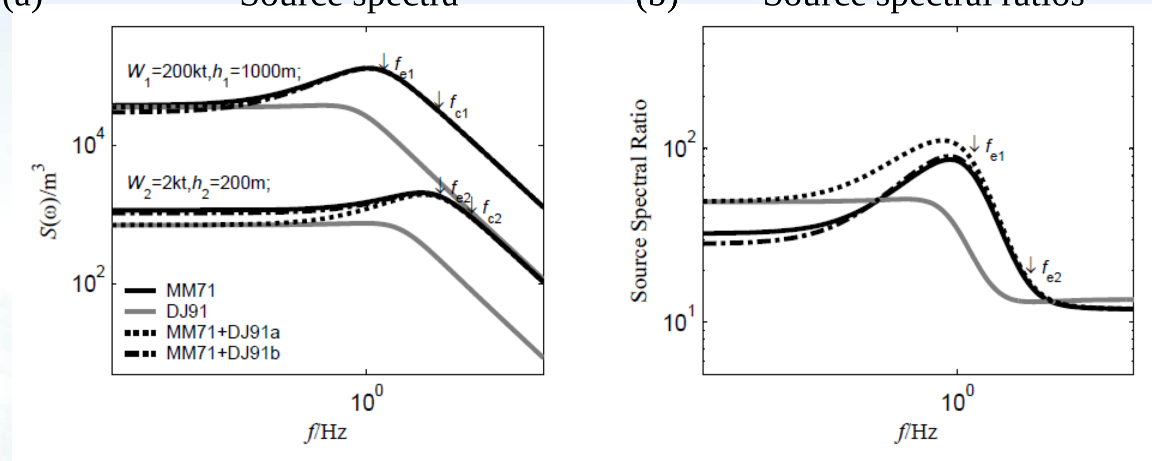


Figure 5. (a) Comparison between source spectra predicted by different source models for two assumed underground explosion of yield W_1=200 kt, W_2=2 kt, and DOB h_1=1000 m, h_2=200 m. Source medium parameters for the computation are ρ_s=2700 kg/m^3,α_s=5000 m/s,ν_s=0.25 and GP=1. (b) Respective source spectral ratios between the assumed explosions .

• pP-wave’s effect on the explosions’ spectral ratios

✓if a pair of explosions have different pP-P time differences, fluctuation due to interference between pP and P wave will appear in the resultant spectral ratio;

✓ The amplitude of the fluctuation depends upon the value of the pP reflection coefficient;

✓the frequency of the first spectral ratio notch is the inverse of the pP-P time difference of the larger explosion, if the spectral ratio is for the larger to the smaller explosion.

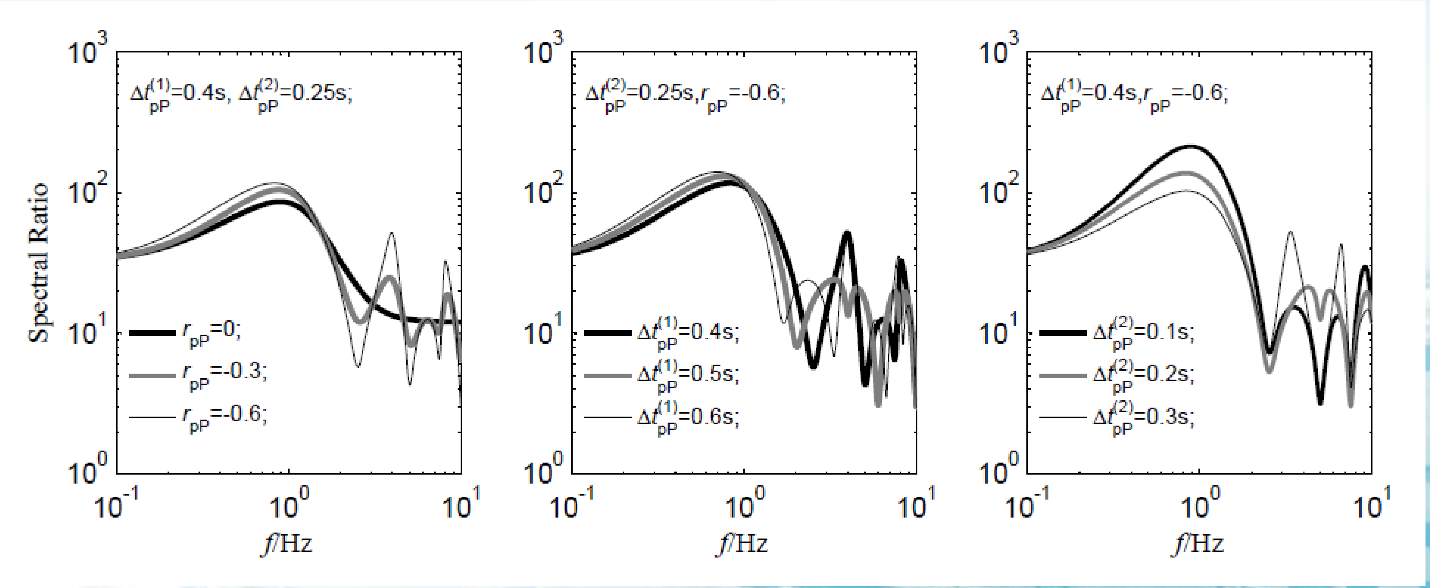


Figure 6 Illustration of pP-wave’s effect on the explosions’ spectral ratios.

3. Empirical modeling of the teleseismic spectral ratios.

- As no pP-wave effect upon regional P-wave spectral ratios is observed among the Korean nuclear tests, we assume r_{pP}=0 in this situation and the observed network-averaged spectral ratio of regional P-wave approximately equal to respective source spectral ratio for a given pair of explosions.
- For this reason, we may use a source-model-independent method to model the observed teleseismic spectral ratios.

Observed spectral ratios

$$SPR_i^{(tele)}(\omega) \approx SPR_i^{(regional)}(\omega) \times \frac{1 + r_{pP}^{(0)} e^{-i\omega\Delta t_{pP}^{(0)}}}{1 + r_{pP}^{(i)} e^{-i\omega\Delta t_{pP}^{(i)}}}$$

$$\varepsilon = \sum_{i=1}^{4} \varepsilon_i \qquad \varepsilon_i(\Delta t_{pP}^{(i)}, r_{pP}^{(i)}; r_{pP}^{(0)}) = \sum_{k=1}^{N_i} r_{ik}^{\ 2}$$

$$r_{ik} = \log_{10} SPR_i^{(\mathrm{TP})}(\omega_k) - \log_{10} SPR_i^{(\mathrm{RP})}(\omega_k) - \log_{10}\left(\frac{1 + r_{pP}^{(0)} e^{-i\omega_k\Delta t_{pP}^{(0)}}}{1 + r_{pP}^{(i)} e^{-i\omega_k\Delta t_{pP}^{(i)}}} \right)$$

(a) (b)

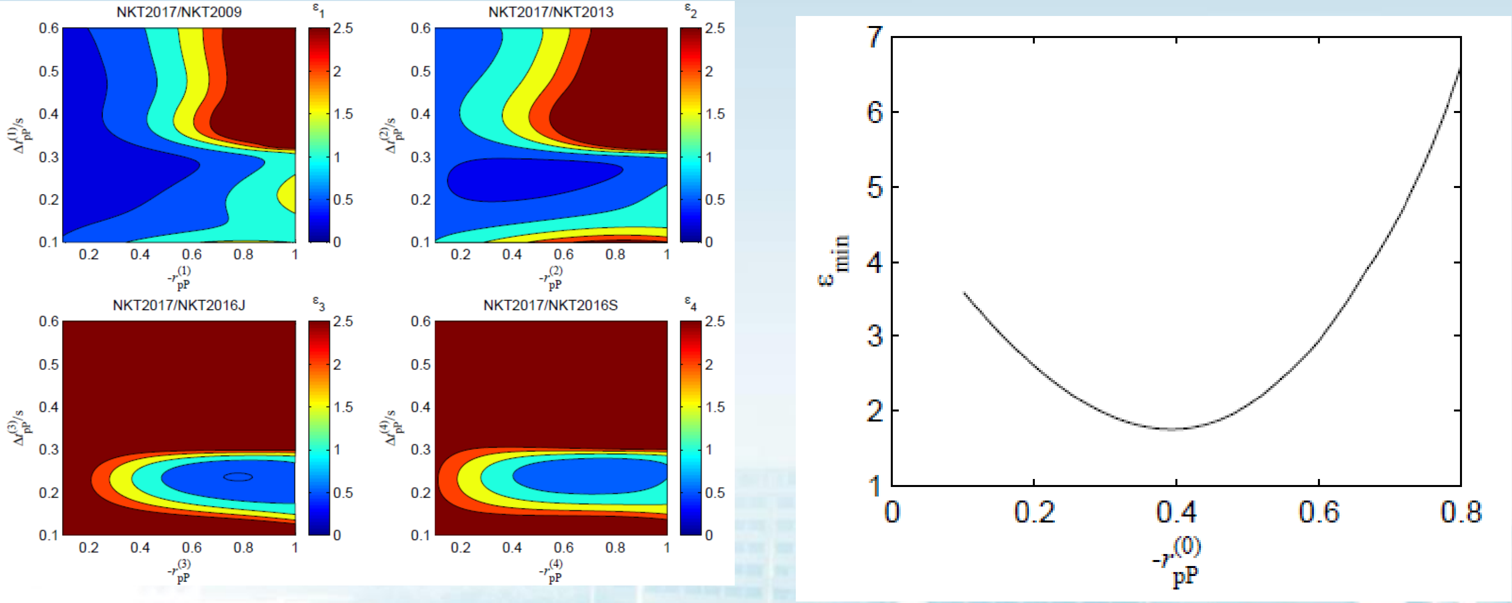


Figure 7. (a) Colored contour plots for individual misfit ε_i's variation against $\Delta t_{pP}^{(i)}$, $r_{pP}^{(i)}$ when $r_{pP}^{(0)}$=-0.42; (b) Minimum total misfit ε_{min} for teleseismic P-wave spectral ratios as a function of NKT2017's pP reflection coefficient $r_{pP}^{(0)}$.

- We solved the pP-wave parameters of the Korean nuclear tests under three different assumptions about the pP reflection coefficients;

Table 2. Estimated pP Parameters under Different Assumptions

Assumptions	Solutions
1(independent r_{pP})	$r_{pP}^{(0)} = -0.42$; $r_{pP}^{(1)} = -0.26$; $r_{pP}^{(2)} = -0.48$; $r_{pP}^{(3)} = -0.78$; $r_{pP}^{(4)} = -0.70$;
	$\Delta t_{pP}^{(1)} = 0.275s$; $\Delta t_{pP}^{(2)} = 0.26s$; $\Delta t_{pP}^{(3)} = 0.235s$; $\Delta t_{pP}^{(4)} = 0.245s$;
2 ($r_{pP}^{(1)} = r_{pP}^{(2)} = r_{pP}^{(2)} = r_{pP}^{(4)} \neq r_{pP}^{(0)}$)	$r_{pP}^{(0)} = -0.45$; $r_{pP}^{(1)} = r_{pP}^{(2)} = r_{pP}^{(3)} = r_{pP}^{(4)} = -0.65$;
	$\Delta t_{pP}^{(1)} = 0.28s$; $\Delta t_{pP}^{(2)} = 0.27s$; $\Delta t_{pP}^{(3)} = 0.24s$; $\Delta t_{pP}^{(4)} = 0.25s$;
3($r_{pP}^{(0)} = r_{pP}^{(1)} = r_{pP}^{(2)} = r_{pP}^{(2)} = r_{pP}^{(4)}$)	$r_{pP}^{(0)} = r_{pP}^{(1)} = r_{pP}^{(2)} = r_{pP}^{(3)} = r_{pP}^{(4)} = -0.5$;
	$\Delta t_{pP}^{(1)} = 0.285s$; $\Delta t_{pP}^{(2)} = 0.265s$; $\Delta t_{pP}^{(3)} = 0.24s$; $\Delta t_{pP}^{(4)} = 0.25s$;

- ✓ The observed notch on teleseismic P-wave spectral ratios may be well modeled with interference between P- and apparent pP-waves of the explosions;
- ✓ The apparent pP-P time delay for NKT2017 is approximately 0.4 s;
- ✓ Apparent pP-P time delay for NKT2009~2016S are between 0.2-0.3 s with the pP-wave reflection coefficients close to that for an elastic pP wave;
- ✓ The apparent pP-wave reflection coefficient for NKT2017 is significantly smaller than that for previous Korean nuclear tests;
- ✓ The estimated pP-wave parameters for NKT2009~NKT2016S are consistent with the common view that the Korean nuclear tests preceding NKT2017 were overburied, causing little degree of surface rock spallation or late-time rock damage.

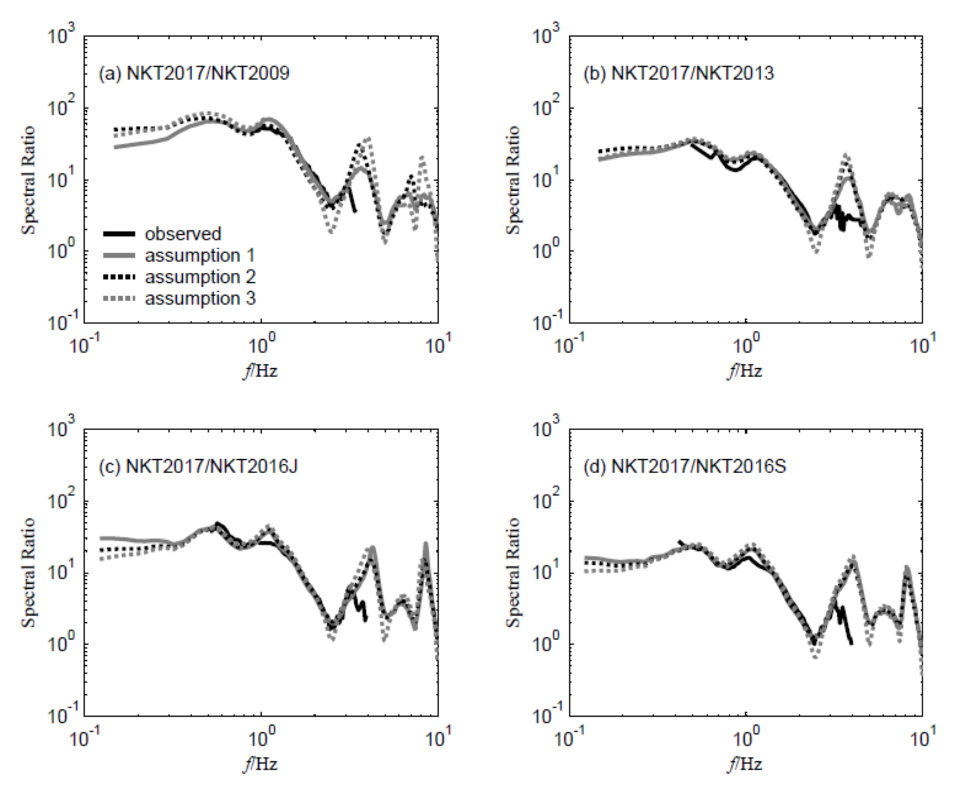


Figure 8. Comparison between observed and semi-synthetic teleseismic P-wave spectral ratios derived by the empirical modeling.

4. Theoretical modeling of the regional P-wave spectral ratios

- We modeled the observed regional P-wave spectral ratios using different explosion source models assuming $r_{\mathrm{pP}}=0$.

$$\varepsilon_i(W_i, h_i; W_0, h_0) = \sum_{k=1}^{N_i} w_{ik} r_{ik}^{\ 2}$$

$$r_{ik} = \log_{10}[S(\omega_k; W_0, h_0) / S(\omega_k; W_i, h_i)] - \log_{10} SPR_i^{(\mathrm{RP})}(\omega_k)$$

$$w_{ik} \propto \log_{10} f_{k+1} - \log_{10} f_{k-1}; \sum_{i=1}^{N_i} w_{ik} = 1$$

- As the absolute yields and DOBs of the explosions cannot be uniquely determined, we attempted to determine W_0, W_i and h_i (i=1,2,3,4) as functions of h_0 by minimizing the total misfit for different values of it.

(a) (b)

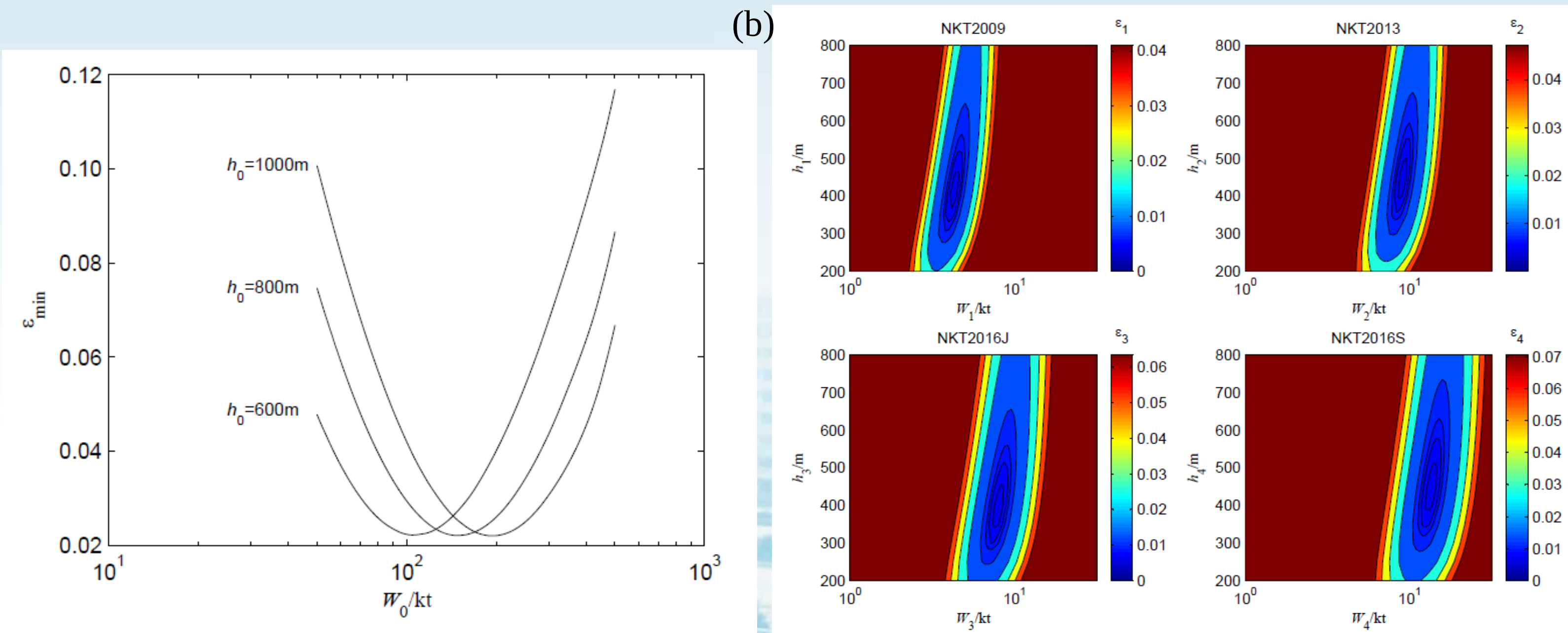


Figure 9. (a) Misfit ε_{min}'s variation against W_0 assuming different h_0 values for model MM71+DJ91a; (b) Colored contour-plot for ε_i's variation against W_i, h_i assuming h_0=800m,W_0=150kt in the case of model MM71+DJ91a.

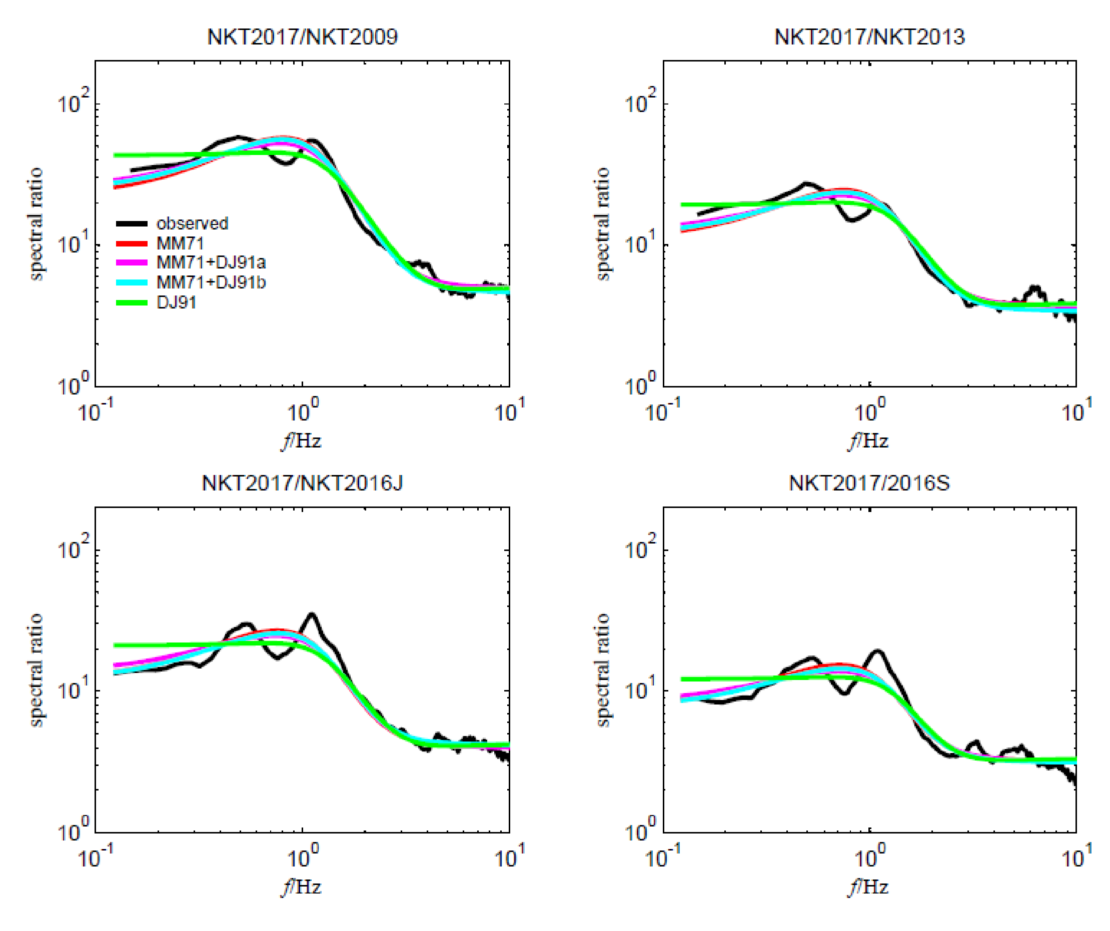


Figure 10. Comparisons between observed and synthetic regional P-wave spectral ratios derived from optimum yield and DOB estimations corresponding to h_0=800m. For a given source model, estimations corresponding to other h_0 values will give identical results.

- Modeling Result 1: Yields of North Korean Nuclear Tests

✓ Yield estimations increases with the h_0 (DOB of NKT2017) ;

✓ For h_0 in the range of 600-1100m, yield estimations by the MM71-related models are approximately 100–300 kt for NKT2017, 3–7 kt for NKT2009, 6–15 kt for NKT2013 and NKT2016J, and 10–25 kt for NKT2016S;

✓ Yield estimations by model DJ91 are significantly smaller than that by MM71-related models;

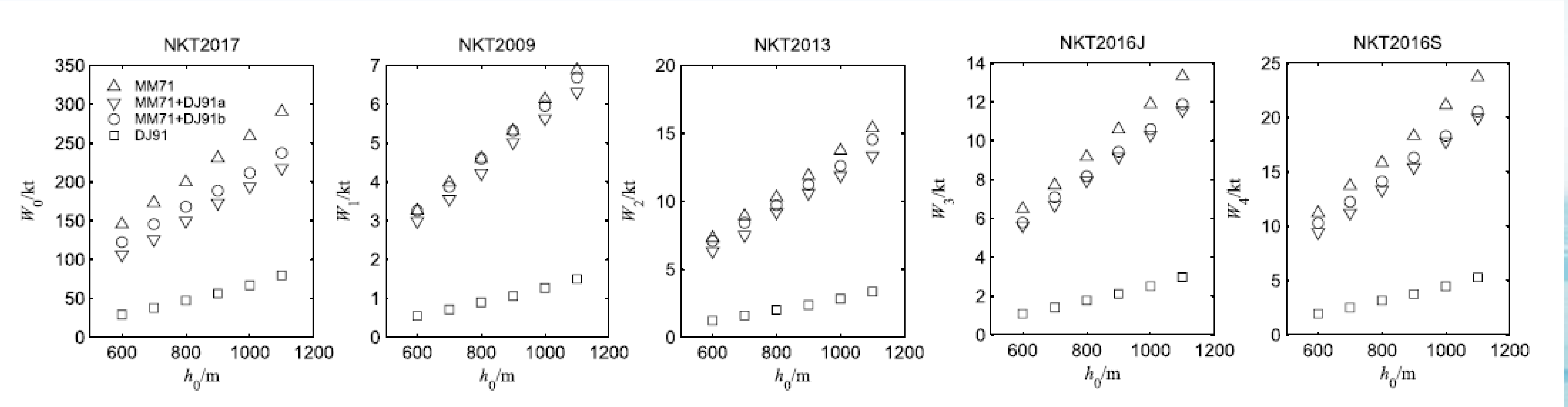


Figure 11. Yield estimations of Korean nuclear tests as functions of NKT2017's DOB by different types of source models.

- ✓ Yield estimations by MM71-related models are basically consistent with that by body wave magnitude. While yield estimations by the DJ91 model are much smaller.

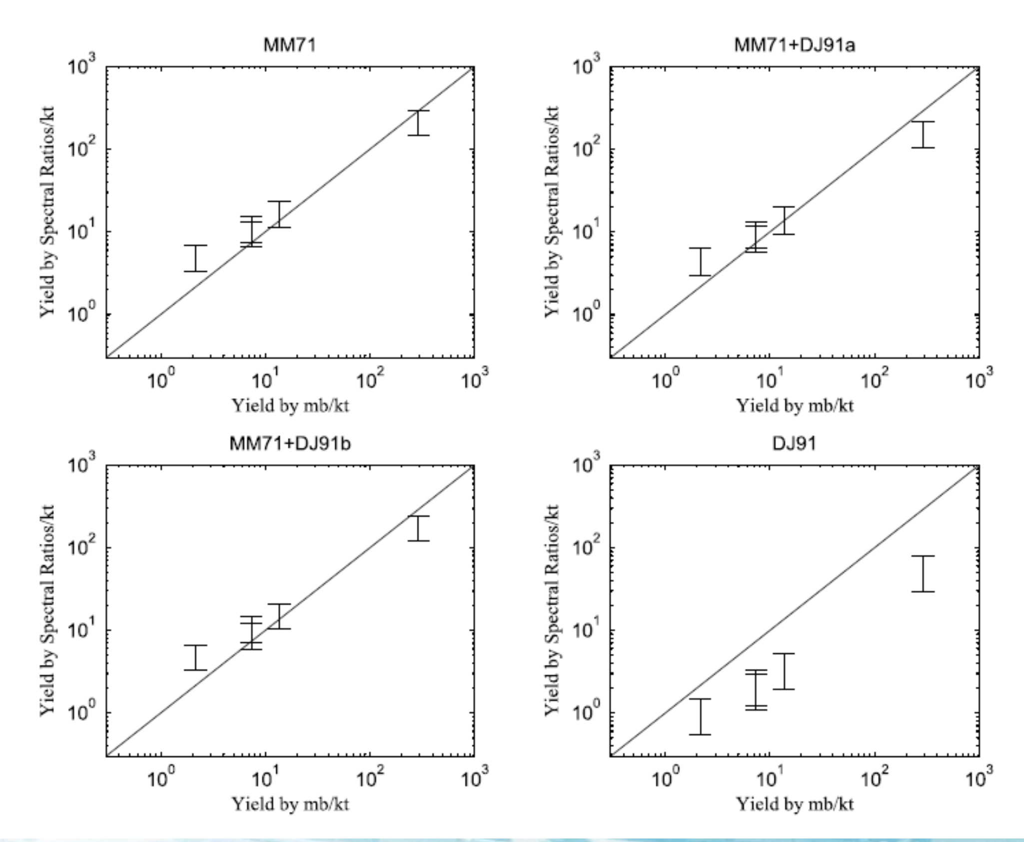


Figure 12. Comparison between the yield estimations by modeling the observed regional P-wave spectral ratios assuming different kind of source models and the yield estimations obtained by mb (NEIC) using the mb/yield relationship mb=4.45+0.75logW (Murphy, 1996).

• Modeling Result 2: DOBs of the Korean Nuclear Tests

✓ Besides yields of the explosions, we have also obtained DOB estimations for NKT2009-2016S as functions of h_0;

✓ DOB estimations for the earlier four tests are always significantly smaller than the DOB of the 2017;

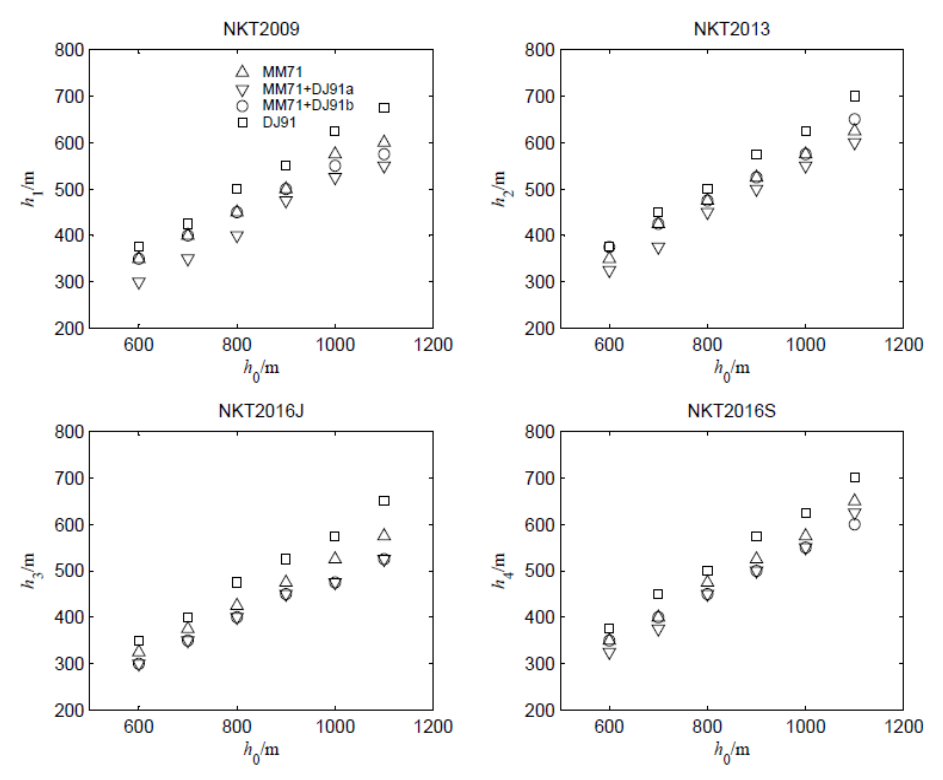


Figure 13. DOB estimations of NKT2009, NKT2013, NKT2016J and NKT2016S as functions of NKT2017's DOB by different models.

- The DOB solutions is of significance as for a given value of h_0, the high-frequency asymptote of the observed spectral ratio between NKT2007 and any other Korean nuclear test cannot be well fitted by too small or too large DOB values of the later.

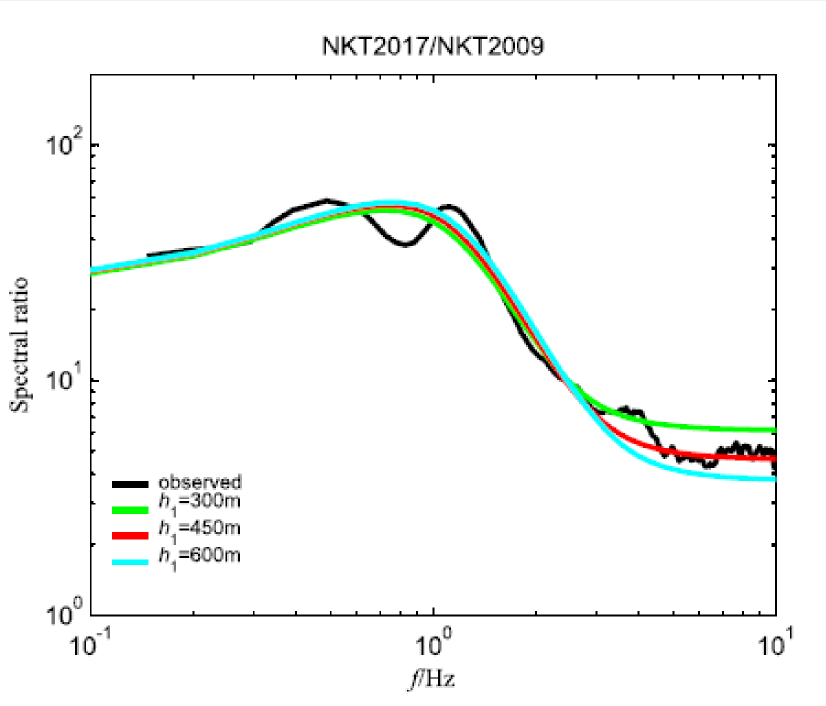


Figure 14. Comparisons between observed and synthetic NKT2017/NKT2009 regional P wave spectral ratios assuming NKT2017's DOB h_0 =800 m and NKT2009's DOB h_1 = 300, 450, and 600 m, respectively.

- Modeling Result 3: Modeling the Lg-wave spectral ratios using results from P-wave spectral ratios.

✓ Observed Lg-wave spectral ratios can also be well modeled using the same yield, DOB estimations as for P wave, but with corresponding source corner frequencies reduced by the near source shear to compressional velocity ratio β_s/α_s.

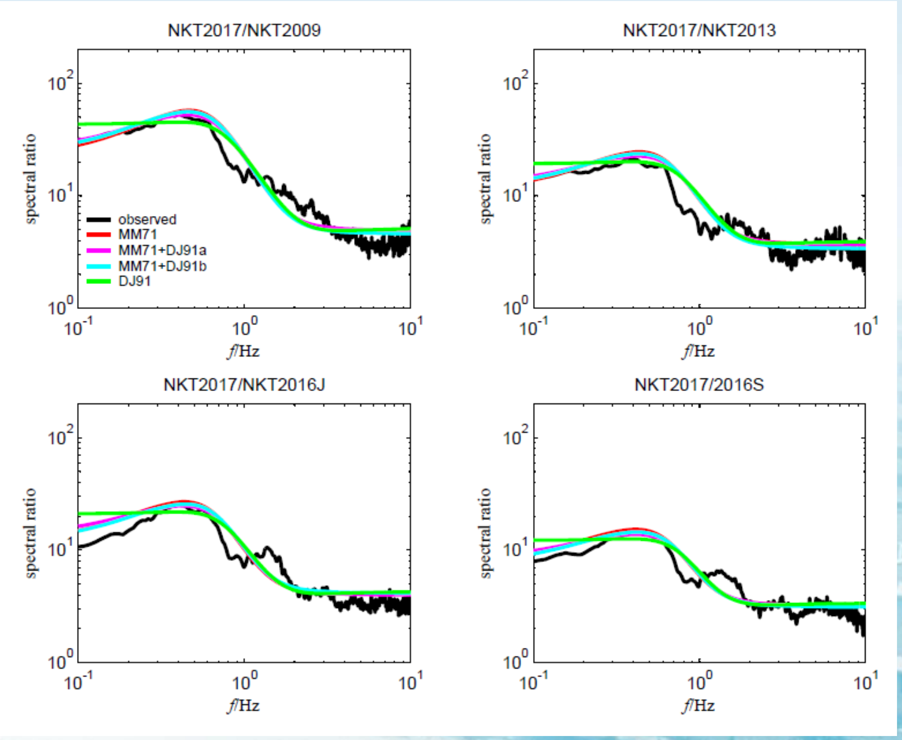


Figure 15. Comparisons between observed and synthetic Lg-wave spectral ratios.

5. Summary

- Teleseismic P-wave spectral ratios between NKT2017 and earlier Korean nuclear tests manifest obvious pP/P interference with a unique notch at approximately 2.5 Hz. The interference effect is absent from spectral ratios observed at regional stations;
- The apparent pP-P time delay for NKT2017 is approximately 0.4s , and between 0.2-0.3s for NKT2009~ 2016S. The significant greater pP-P time delay of NKT2017 may be caused either by that NKT2017 has a greater DOB, or alternatively, by that it experienced a significantly stronger surface spallation;
- The observed Lg-wave spectral ratios of Korean nuclear tests are similar to those of regional P-wave with the explosions' source corner frequencies significantly reduced, supporting the conjecture by Fisk(2006);

- The observed spectral ratios of Korean nuclear tests can be well fitted using MM71-related explosion source models. For the range of 600~1100 m for NKT2017's DOB h_0 , yield estimations by the modeling are approximately 100-300 kt for NKT2017, 3-7 kt for NKT2009, 6-15 kt for NKT2013 and NKT2016J, and 10-25 kt for NKT2016S.
- DOB estimations of NKT2009–2016S obtained via modeling the regional P wave spectral ratios are always significantly smaller than NKT2017's DOB h_0. Whether this is the reflection of the true difference in the explosions' real DOBs or it is just caused by the incompleteness of the model is an open question.

Thanks for your attention.

For details of this study, please refer to:

Jin et al. (2019) Seismic Spectral Ratios between North Korean Nuclear Tests: Implications for their Seismic Sources, J. Geophys. Res.—Solid Earth, DOI: 10.1029/2018JB016554.