INSTITUT
DE RADIOPROTECTION
ET DE SÛRETÉ NUCLÉAIRE

Enhancing nuclear safety

A Methodology to Establish Bayesian Detection Limits for Radionuclide Monitoring

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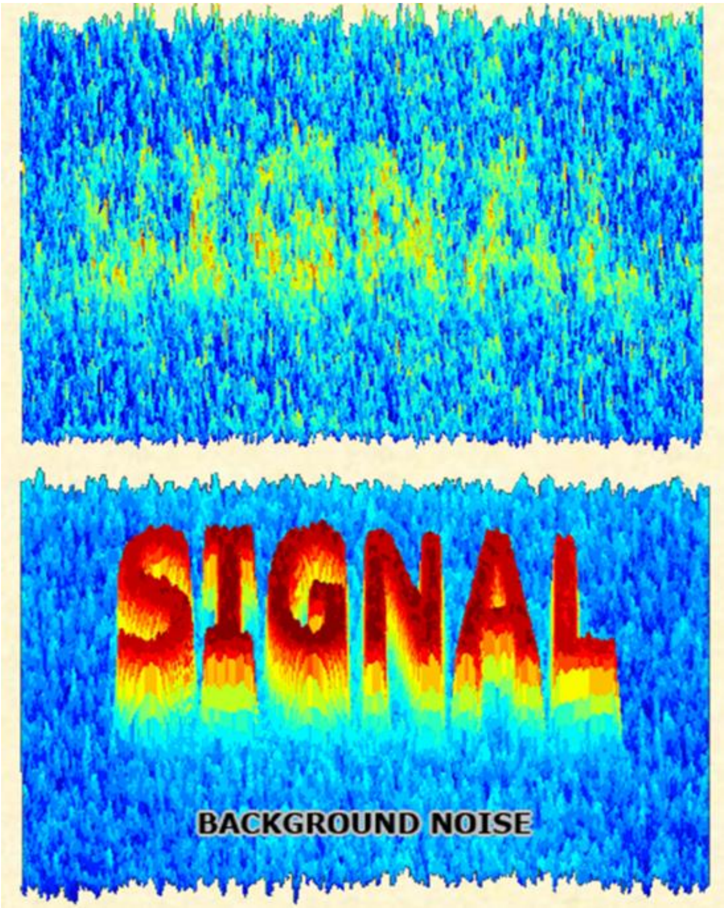

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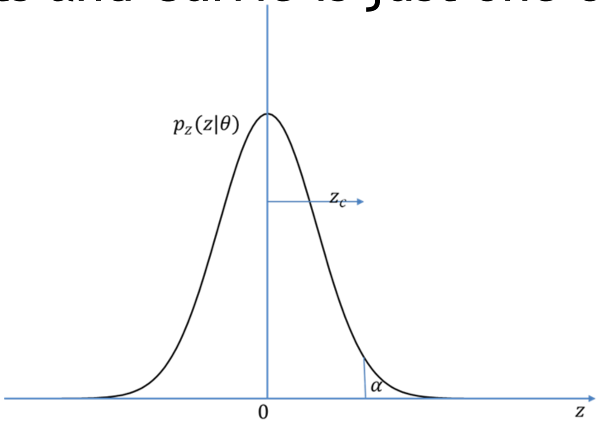
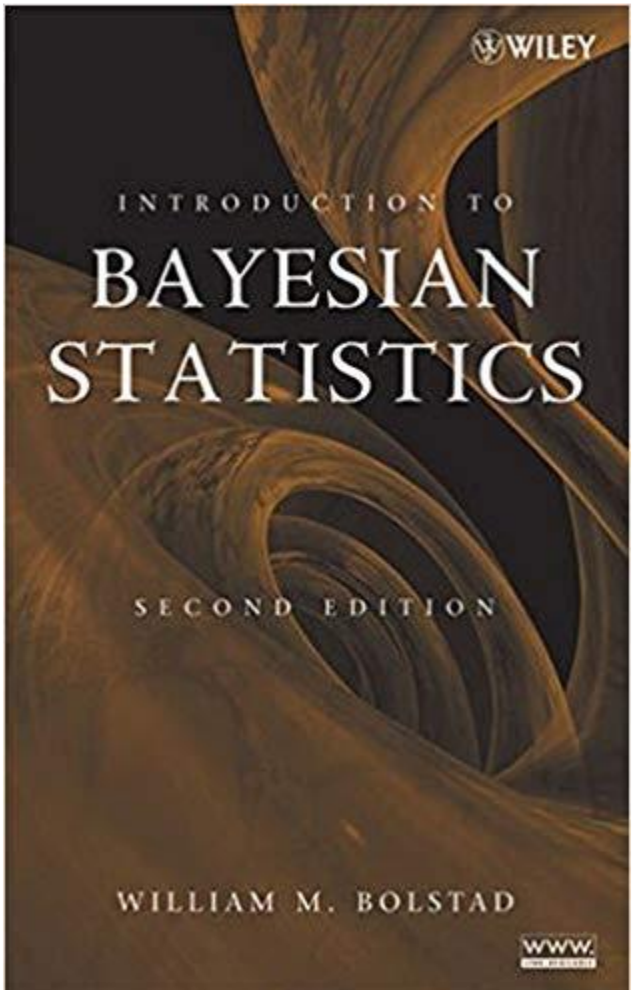
- We consider the situation where we have determined a baseline (blank, background noise without any signal) and we measure a sample where a radionuclide might be present

- The decision that a radionuclide is present is usually made using <u>A</u> frequentist hypothesis test (Currie 1968).

- You have several possible hypothesis tests and Currie is just one of them. It is not THE hypothesis test.

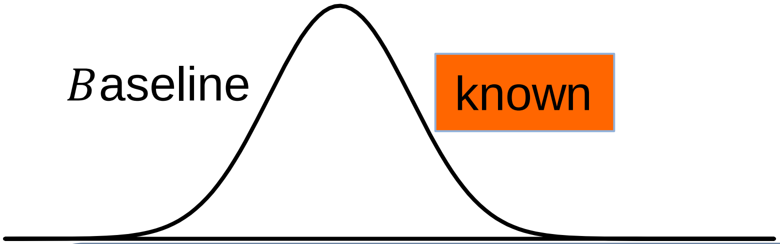


Credible intervals as an hypothesis test



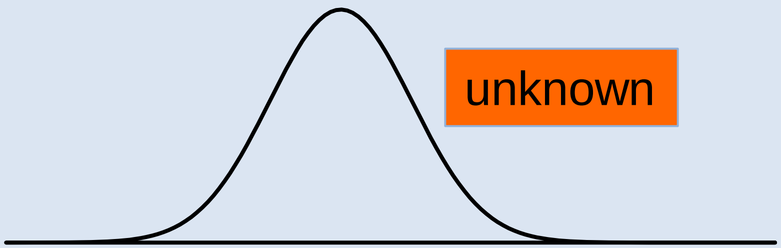
« If $\boldsymbol{\mu_o}$ lies inside the credible interval, we conclude that $\boldsymbol{\mu_o}$ still has credibility as a possible value. In that case we will not reject the null hypothesis : $\boldsymbol{\mu} = \boldsymbol{\mu_o}$, so we consider that it is credible that there is no effect.
However, if $\boldsymbol{\mu_o}$ lies outside the credible interval, we conclude that $\boldsymbol{\mu_o}$ does not have credibility as a possible value, and we will reject the null hypothesis. »

- A result is significant if we can reject the null hypothesis of zero value of the parameter i.e. if 0 is not in the credible interval. 0 is not a plausible value



$b(\mu_b|x)$

$mean\ \hat{b}$
$standard\ error\ \sigma_b$



Signal

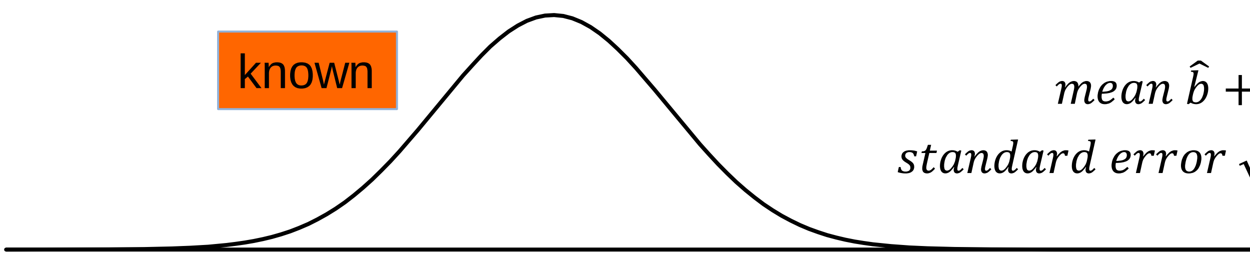
$s(\mu_s|x)$

$mean\ \hat{s}$
$standard\ error\ \sigma_s$

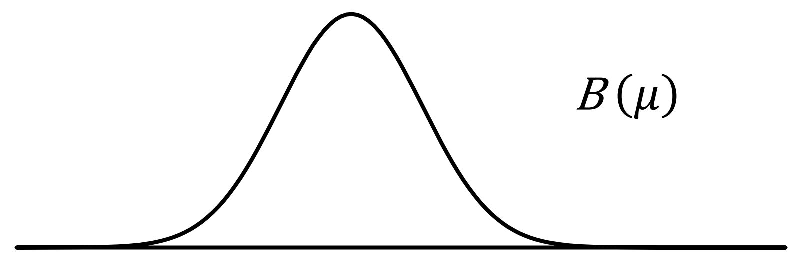
convolution

$$e(\mu) = (b * s)(\mu) = \int_{-\infty}^{\infty} b(y-\mu).s(y)dy$$

$Gross = Baseline + Signal$



$mean\ \hat{b} + \hat{s}$
$standard\ error\ \sqrt{\sigma_b{}^2 + \sigma_s{}^2}$



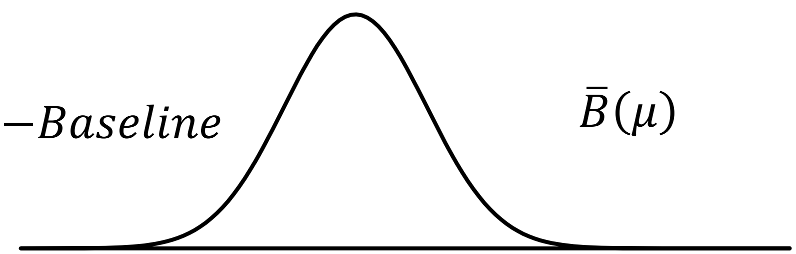
$B(\mu)$

$mean\ \hat{\beta}$

$standard\ error\ \sigma_b$

$Baseline$

$-Baseline$



$\bar{B}(\mu)$

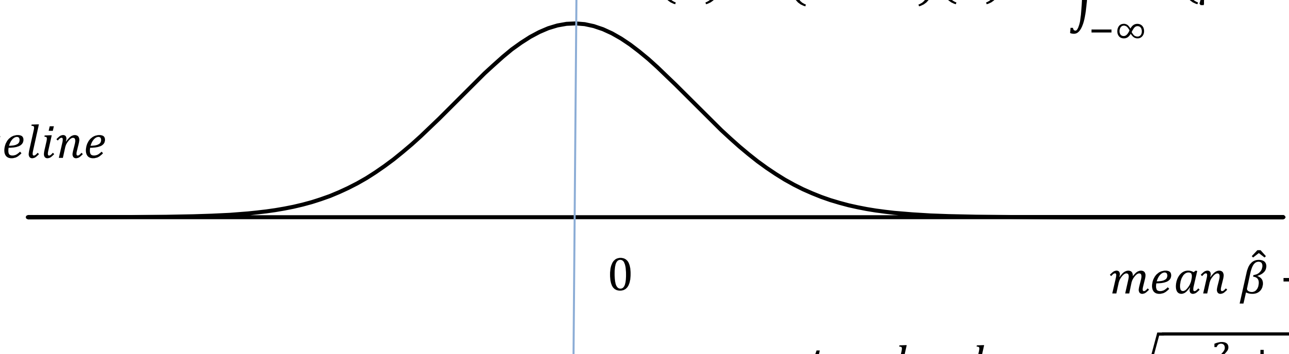
$mean\ \ -\hat{\beta}$

$standard\ error\ \sigma_b$

cross correlation

$$h(\delta) = (B * \bar{B})(\delta) = \int_{-\infty}^{\infty} B(\mu+\delta).B(\mu)d\mu$$

$Baseline - Baseline$



0

$mean\ \hat{\beta} - \hat{\beta} = 0$

$standard\ error\ \sqrt{{\sigma_b}^2 + {\sigma_b}^2} = \sqrt{2{\sigma_b}^2}$

You can have negative value because of the uncertainty on the exact position of the baseline

$$\beta(\mu|x_b) = e^{-(x_b-\mu)^2/2\sigma_b^2}$$

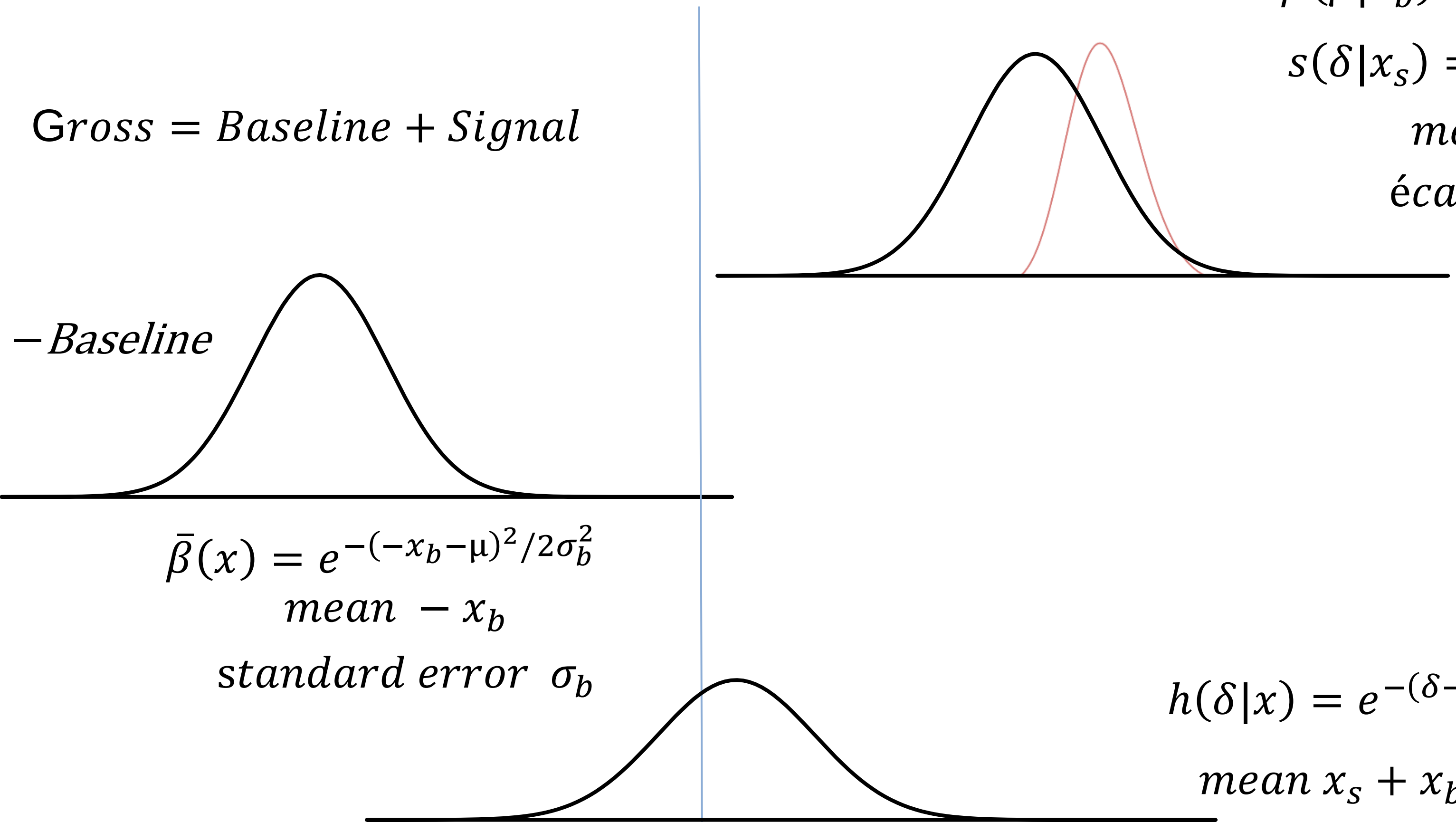
$$s(\delta|x_s) = e^{-(x_s-\delta)^2/2\sigma_s^2}$$

$mean\ x_b + x_s$

$\acute{e}cart\ type\sqrt{{\sigma_b}^2 + {\sigma_s}^2}$

$\text{Gross} = Baseline + Signal$

$-Baseline$



$$\bar{\beta}(x) = e^{-(-x_b-\mu)^2/2\sigma_b^2}$$

$mean\ \ -x_b$

$standard\ error\ \ \sigma_b$

$$h(\delta|x) = e^{-(\delta-x_s)^2/2(2\sigma_b^2+\sigma_s^2)}$$

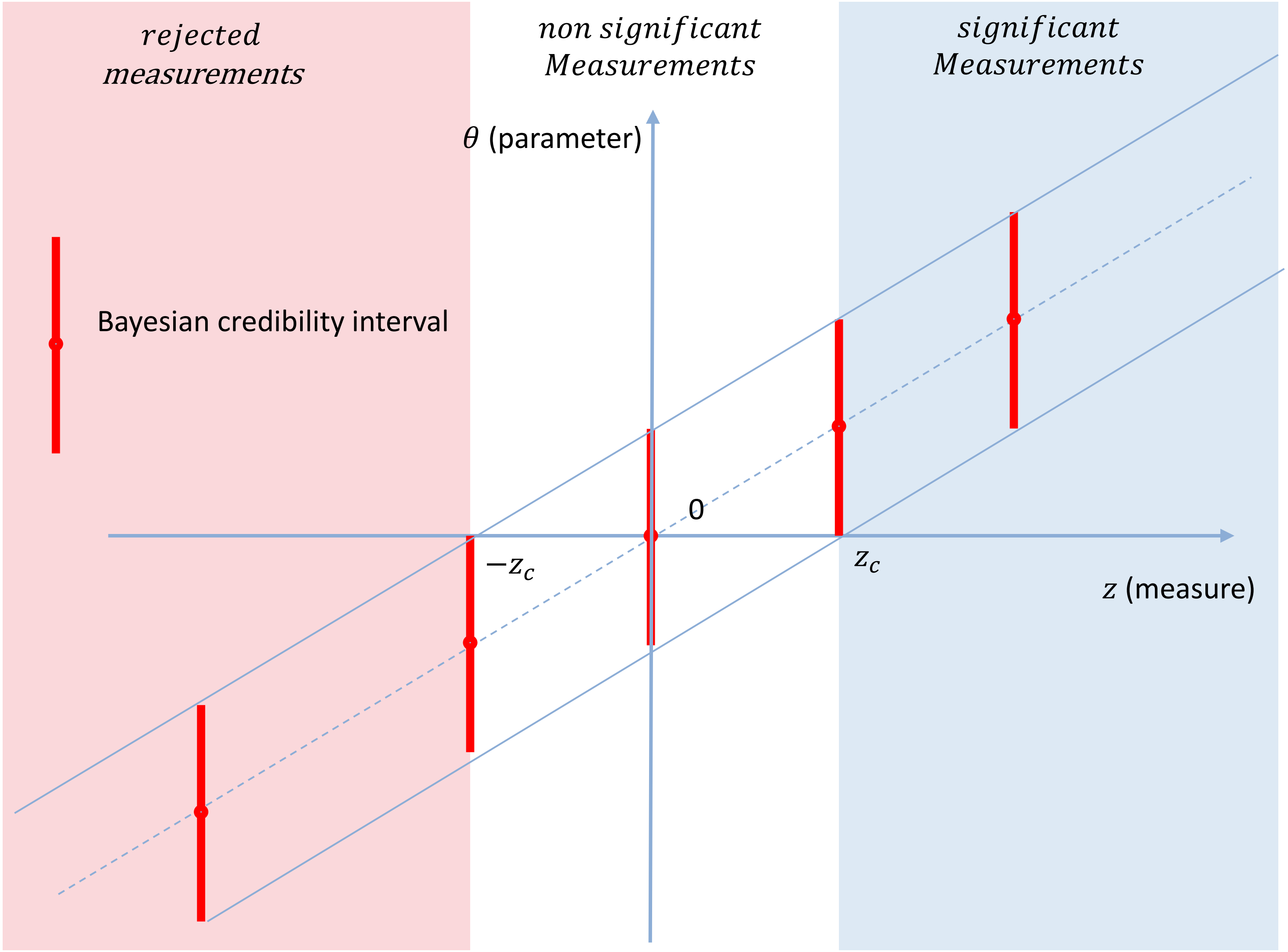
$mean\ x_s + x_b - x_b = x_s$

$standard\ error\ \sqrt{2{\sigma_b}^2 + {\sigma_s}^2}$

0

$Net = Baseline + Signal - Baseline$

Even with a perfectly positive signal, the net value can be negative. It does not mean That the signal is negative just that you have uncertainties on the baseline



Decision threshold= lower limit of credibility interval reaches 0=100% uncertainty
Detection limit= upper limit of credibility interval at the decision threshold

Control by Simulations

- Testing the presence of zero in confidence intervals for homoscedastic distributions gives exactly the same results as the currie tests.

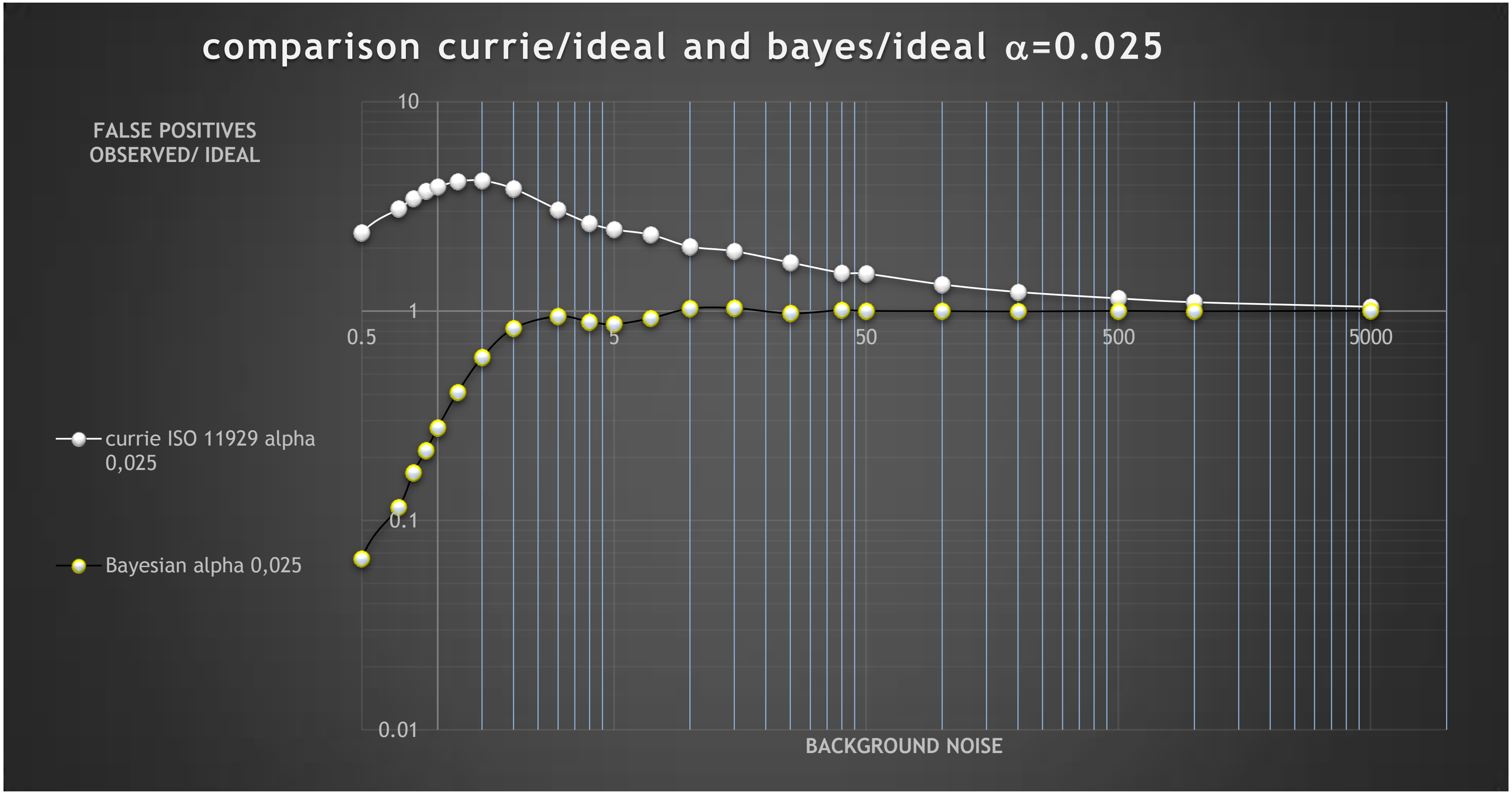
- In order to simulate heteroscedastic distributions, we use Poisson distribution (inspired by STROM 2001). We draw a sample of this poisson distribution. From this sample we determine the decision threshold Lc both from currie and our bayesian theory at a given level of confidence α .

- We draw another sample x and we compare it to Lc

- We repeat one million times. Any significant result is a false positive. We should get α% rate of false positive.

- In essence we use the noise to determine the decision threshold, and we test if the same noise is considered significant

Simulation results



Use of the bayesian credible interval gives much better decision thresholds than currie

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Conclusions

- The determination of the credible interval provides enough information to decide if a result is significant or not, to know the decision threshold, the detection limit and to provide an upper limit for non significant result. This has been pointed out by many authors

- Interval estimation is a form of statistical hypothesis testing and its performance is normally much better than the traditionnal Currie test.

- From a frequentist point of view this better performance can be proven with the use of Neyman-Pearson lemma

1. Summary: from a net measurement x, determine a credible interval [a,b] without excluding negative value.

2. If 0 is in the interval, the result is non significant.

3. You always have an upper limit b even for non significant results

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Thank you for your attention

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