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PREPARATORY COMMISSION

Peak identification in EDS Measurements using multiple subset sum problem formulation

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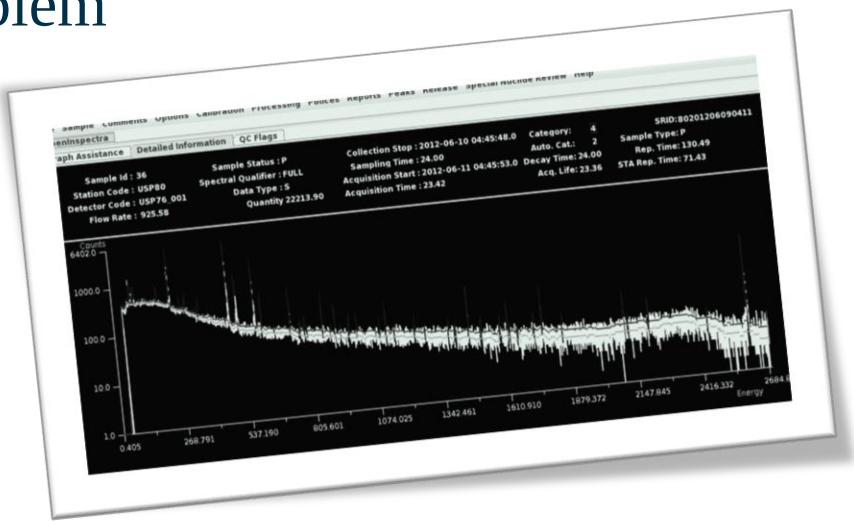
Outline

- Context and Background
- Subset-Sum Problem - SSP
- Gamma Spectrometry
- Unassociated Peak Vs Radionuclide
- Solving the Peak identification Problem
- Summary



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- In the CTBT context, an "event" is any physical occurrence that is registered by the International Monitoring System, be it
  - ✓ *a natural or a man made event,*
  - ✓ *an earthquake or an explosion,*
  - ✓ *a chemical or a nuclear explosion.*

With regards to determining the nature of an event, Members States make the final judgement.

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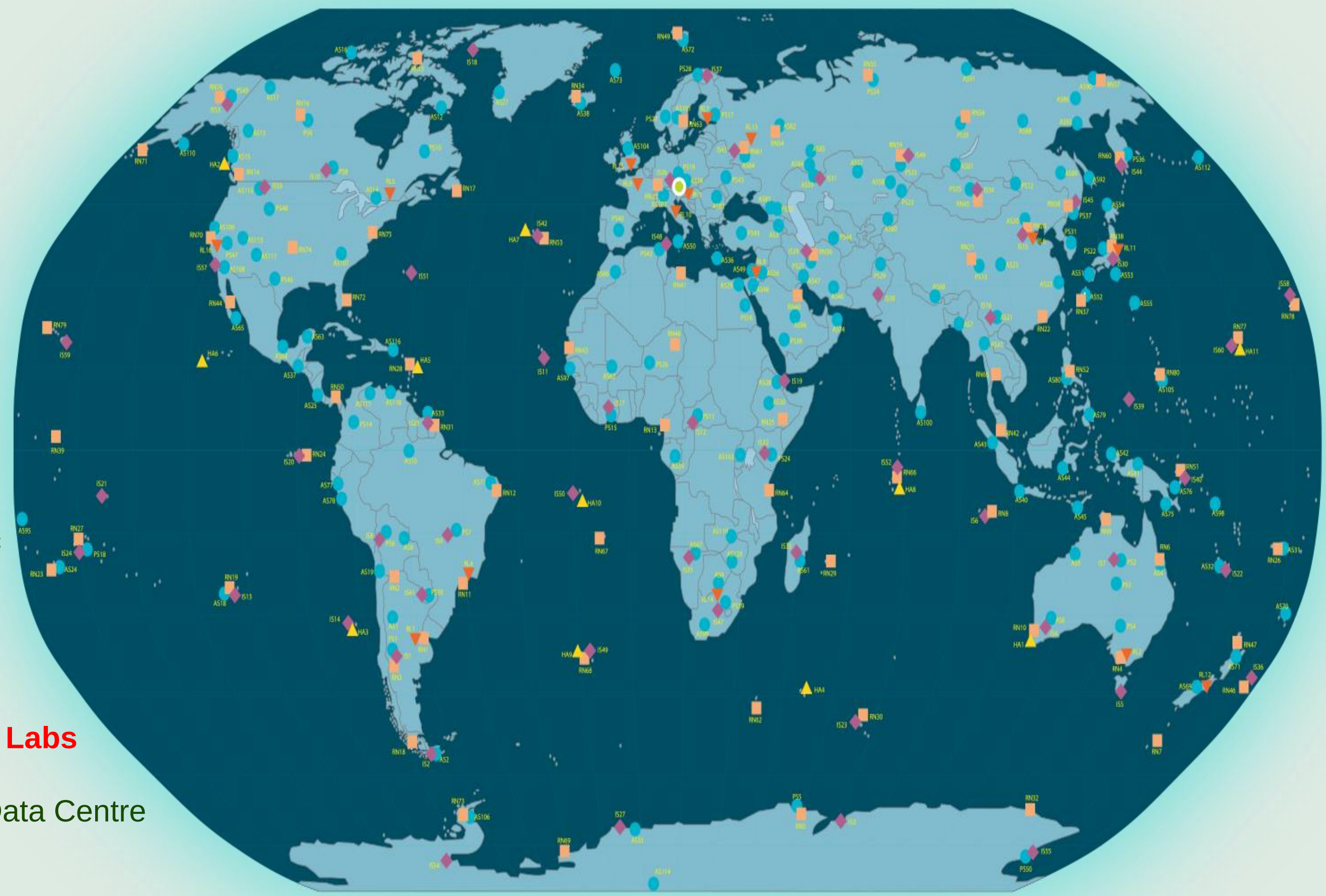
International Monitoring System: 337 Facilities



- Seismic
- Hydroacoustic
- Infrasound
- **Radionuclide**
- **Radionuclide Labs**
- International Data Centre

- Waveform data help identify the location of any event and qualify it as either natural or potentially man-made.
- However, they cannot reliably answer the question as to whether a man-made event, such as an explosion, was nuclear or not. The only reliable way to answer this question is to trace and **analyse** the radioactive remains of a potential nuclear explosion.
- Some remains provide the ultimate proof for the nuclear nature of an event
- Radionuclide monitoring data from the 80 radionuclide monitoring stations worldwide are expected to shed light on this key question.
- Data on radionuclide observations are sent to the International Data Centre (IDC), where they undergo an analysis process.
- After the automatic analysis process, analysts refine the results during interactive review.

- Only the analysis of radionuclide data can provide the ultimate proof to establish the nuclear nature of an explosion
- Each radionuclide particulate monitoring station sends one gamma ray spectrum per day showing which radionuclide were observed in a single sample and in what quantity.
- Each radionuclide Sample was examined and categorized in the corresponding group according to the nature of the radionuclide contained in them.

- In the spectrum analysis process, doubts about the existence of level 4 or 5 nuclides are made.
- In order to ensure that a **suspect nuclide** exists or not, this presentation addresses the problem of **association of unidentified peaks with suspect nuclides**.
- We assume that the corresponding energy is an amount of **sum of energies** provided by the **suspects nuclides**.

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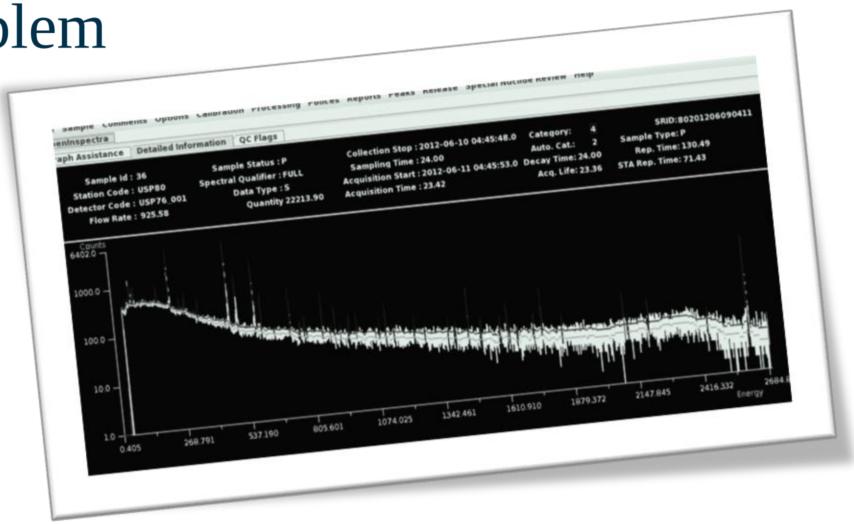

- The Subset Sum Problem (SSP) is an important problem in computer science and combinatorial optimization.
- The challenge is to determine if there is some subset of numbers in an array that can sum up to some number ***s***.
- SSP is to find subset of elements that are selected from a given set ***S*** whose sum adds up to a given number ***K***, where we consider that the set ***S*** contains non-negative values.

Given :

$\boldsymbol{A = (a_1, a_2, \ldots, a_n)}$ and

a positive integer **s**, called the sum.

The question is then : Is there a subset ***A'*** of ***A*** with

$\boldsymbol{SUM_{a' \text{ in } A'}(a') = s}$, or equivalent:

Does there exist a vector

$\mathbf{X = (x_1, x_2, \ldots, x_n), x_i \text{ in } \{0, 1\}, \text{ with } AX = s?}$

If a solution is desired, the problem becomes a **functional problem**:

The functional problem of any vector ***A*** together with a sum ***s*** is denoted by **(A, s).**

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Input Data

- A = (15, 22, 14, 26, 32, 9, 16, 8)
- s = 53.

Solution

(A, 53) is given by the vector X = (1, 1, 0, 0, 0, 0, 1, 0), because

$AX = a_1 + a_2 + a_7$

$= 15 + 22 + 16 = 53.$

The corresponding subset is A' = (15, 22, 16).

it's not the only solution

- Generally, there may exist several solutions to (A, s)
- Another solution for (A, 53) is X = (0, 1, 1, 0, 0, 1, 0, 1) with the subset A' = (22, 14, 9, 8).

- The SSP is in the NP complexity class.
- The decision problem is NP-complete and the corresponding functional problem is NP-hard.
- No polynomial algorithm is known solving the general subset sum problem.
- The trivial possible solution vector for a given (A,s) is :

for all possible 0-1 vectors $X = (x_1, ..., x_n)$ the sum $s' = AX$ is computed.

But, this algorithm is computing the sum of all ***2^n*** different ***X*** vectors, and has an exponential running time of ***$O(2^n)$.***

Meet-in-the-middle algorithm

- One of the more efficient algorithms;
- Recursive algorithm;
- At each iteration, this algorithm divides the original input vector A in two equal parts $A_1 = (a_1, ..., a_t)$ and $A_2 = (a_{t+1}, ..., a_n)$.
- All possible sums $s_1 = A_1X_1$ are computed in a pre-computation step.

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Meet-in-the-middle algorithm

- Input: a vector $A = (a_1, ..., a_n)$ and a sum s.
- Output: a vector X with $AX = s$, provided a solution exists.

I- Divide $A = (a_1, ..., a_n)$ into $A_1 = (a_1, ..., a_t)$ and $A_2 = (a_{t+1}, ..., a_n)$.

For every vector $X_1 = (x_1, ..., x_t)$

- ➢ compute $s_1 = A_1X_1$
- ➢ store s_1 together with the corresponding ***x-vector*** in a table ***T***
- ➢ sort this table by the sum

II- Now for all possible vectors $X_2 = (x_{t+1}, ..., x_n)$

- ➢ compute $s_2 = A_2X_2$
- ➢ get the difference $L = s - s_2$
- ➢ search L in T. If found, a solution vector is $X = X_1||X_2$.

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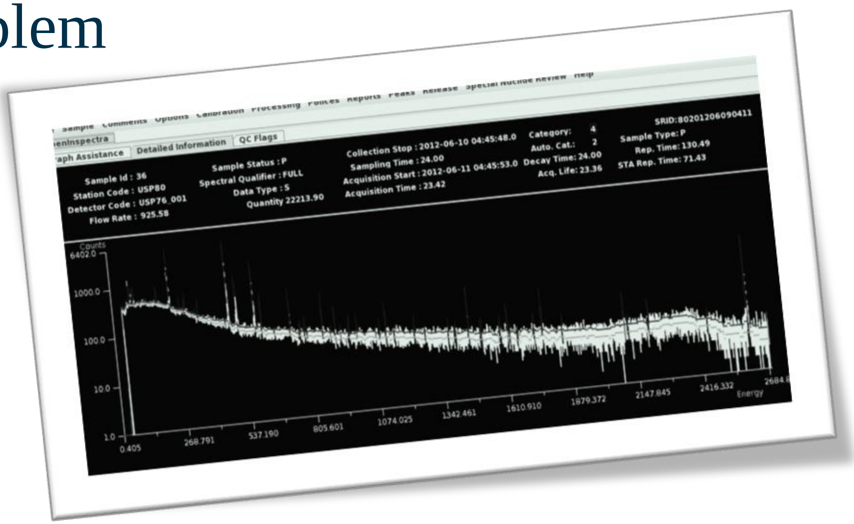
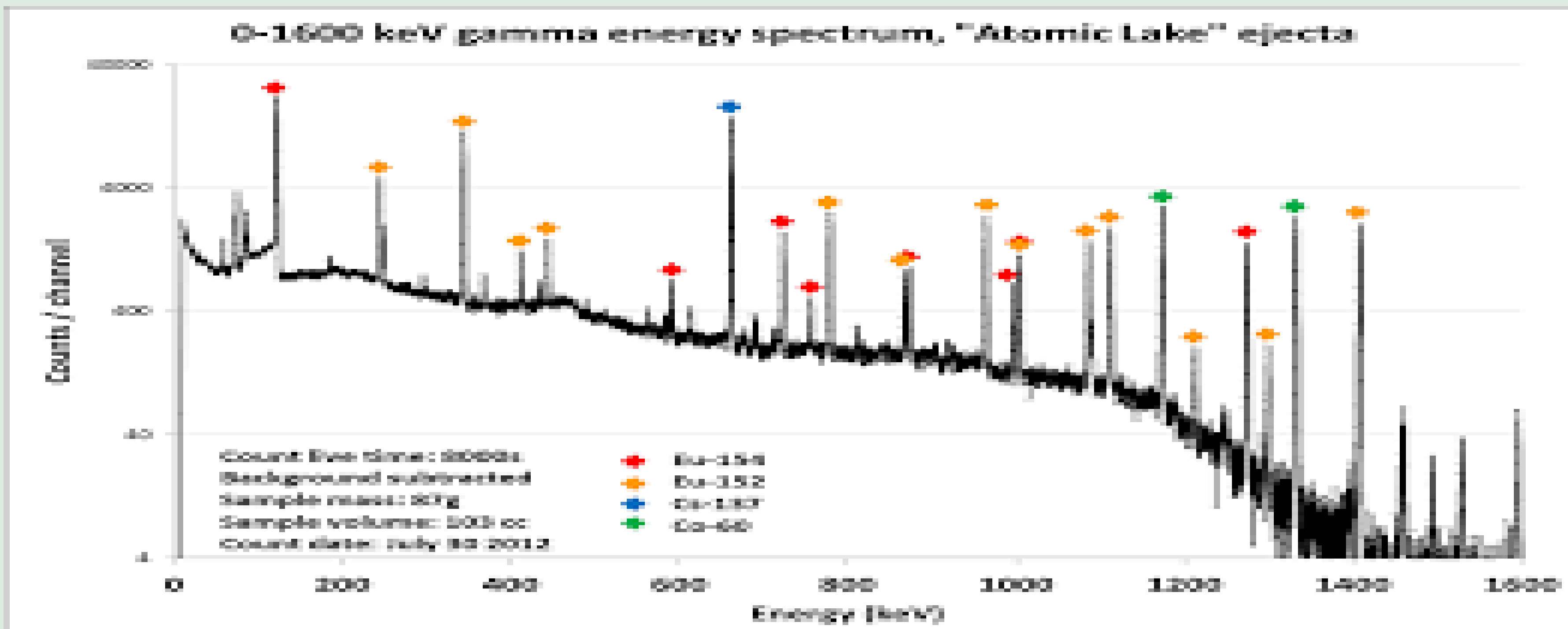
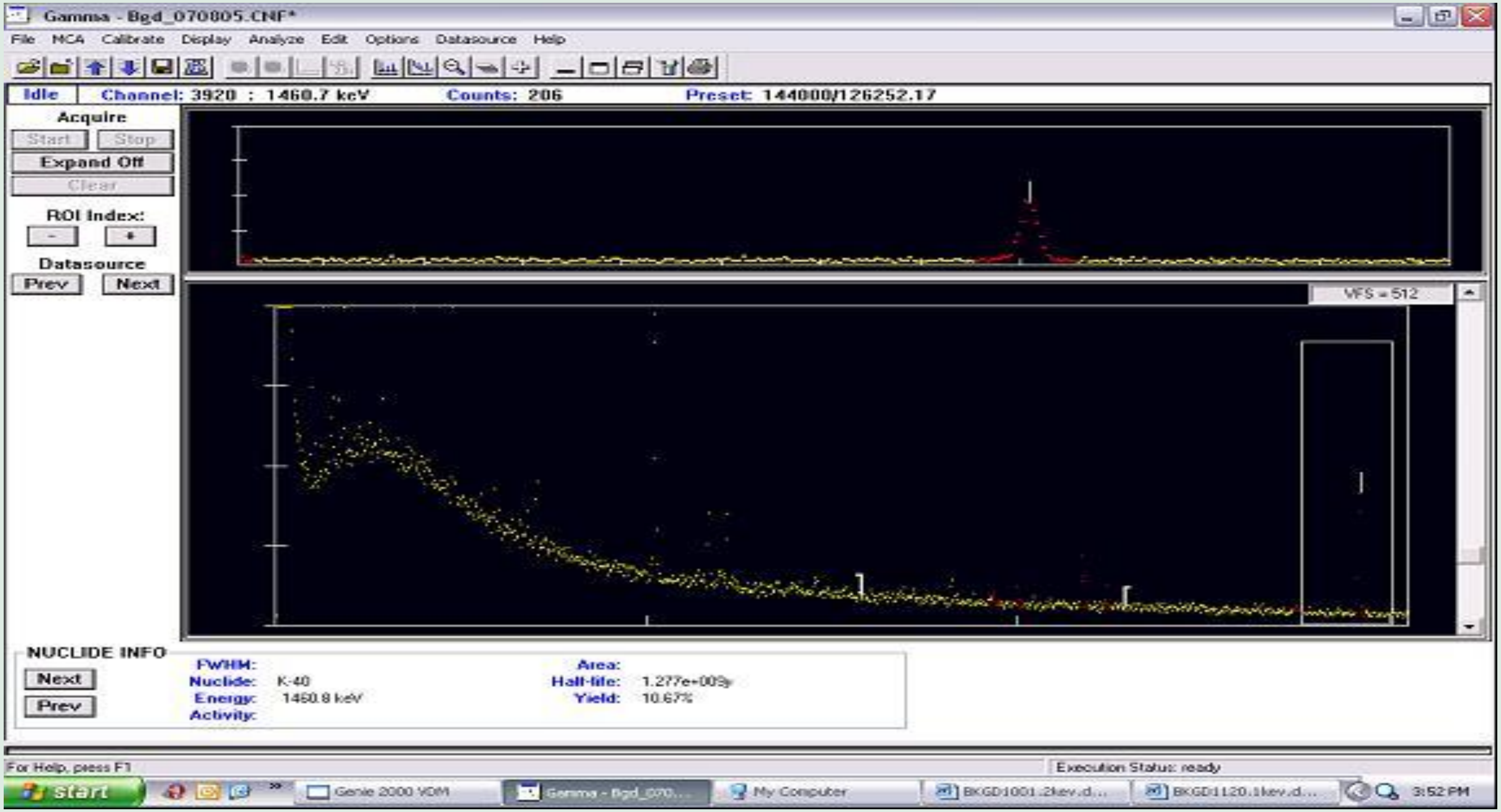


- Most radioactive sources produce gamma rays, which are of various energies and intensities. When these emissions are detected and analyzed with a spectroscopy system, a gamma-ray energy spectrum can be produced.



- Gamma spectrometry is one of the fundamental measuring techniques in nuclear technology for unambiguous, clear identification of radionuclide, even in mixtures of various isotopes.
- The experiment is intended to show the basics of the methodology of gamma spectrometry, the properties of most important gamma detectors and how to identify clearly radionuclides.
- Moreover, the absolute activity of a radionuclide could be determined by means of a gamma spectrometer if it is calibrated absolutely with regards to its efficiency.

- For an IDC or NDCs Analyst, the Analysis of the identity of gamma-emitting radionuclides and measurement of their respective activity in a sample are the important tasks



Peak identification using SSP formulation

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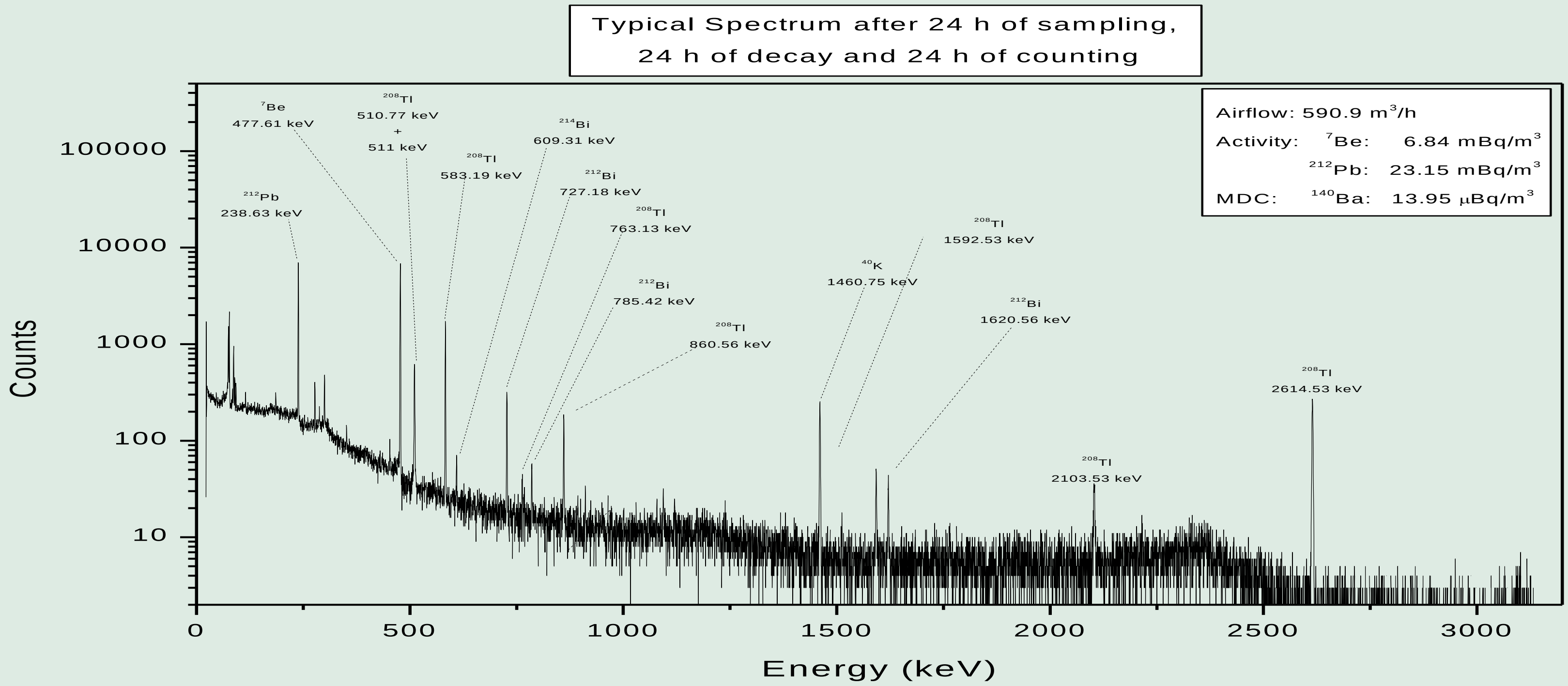
Energy

Gamma emitters can be identified by their respective energies

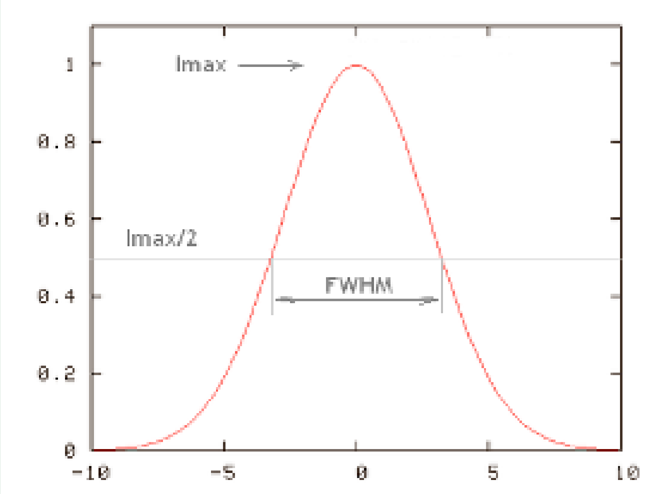
Important characteristic for detection – very low energy gammas are difficult to detect; gamma emitters with similar energies are difficult to identify



Detectors characteristics

Resolution

- ability to separate two gamma peaks that are close to each other in order to be able to separately measure and identify these peaks.
- usually measured with the full-width at half maximum (FWHM). This is the width of the gamma ray peak at half of the highest point on the peak distribution.
- A requirement for gamma detectors used in radionuclide stations is that the FWHM should be <2.5 keV at 1332 keV (Co-60 peak)



Efficiency

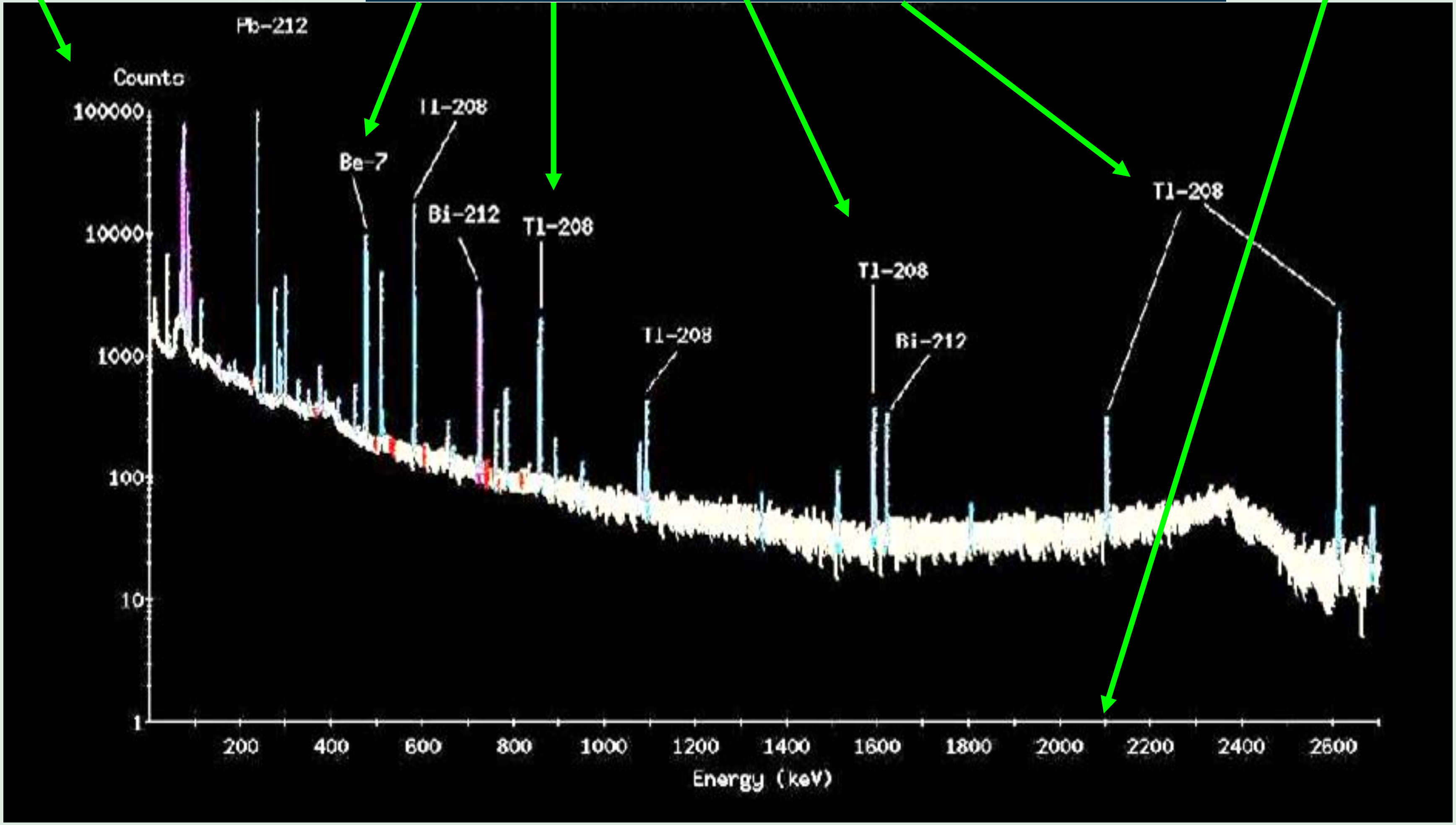
- Relative efficiency values are often used for germanium detectors, and compare the efficiency of the detector at 1332 MeV to that of a 3 in × 3 in NaI detector (another gamma detector). This type of detector was popular and well characterized before the introduction of the germanium detectors.
- Relative efficiency of detectors used at radionuclide stations should be ≥ 40%. The higher the efficiency, the better the detection sensitivity.

Example of station sample spectra

Number of γ-photons collected in the detector

What energy do they have?

What radionuclides and how much are in the sample?



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Software for interactive review : OpenSpectra

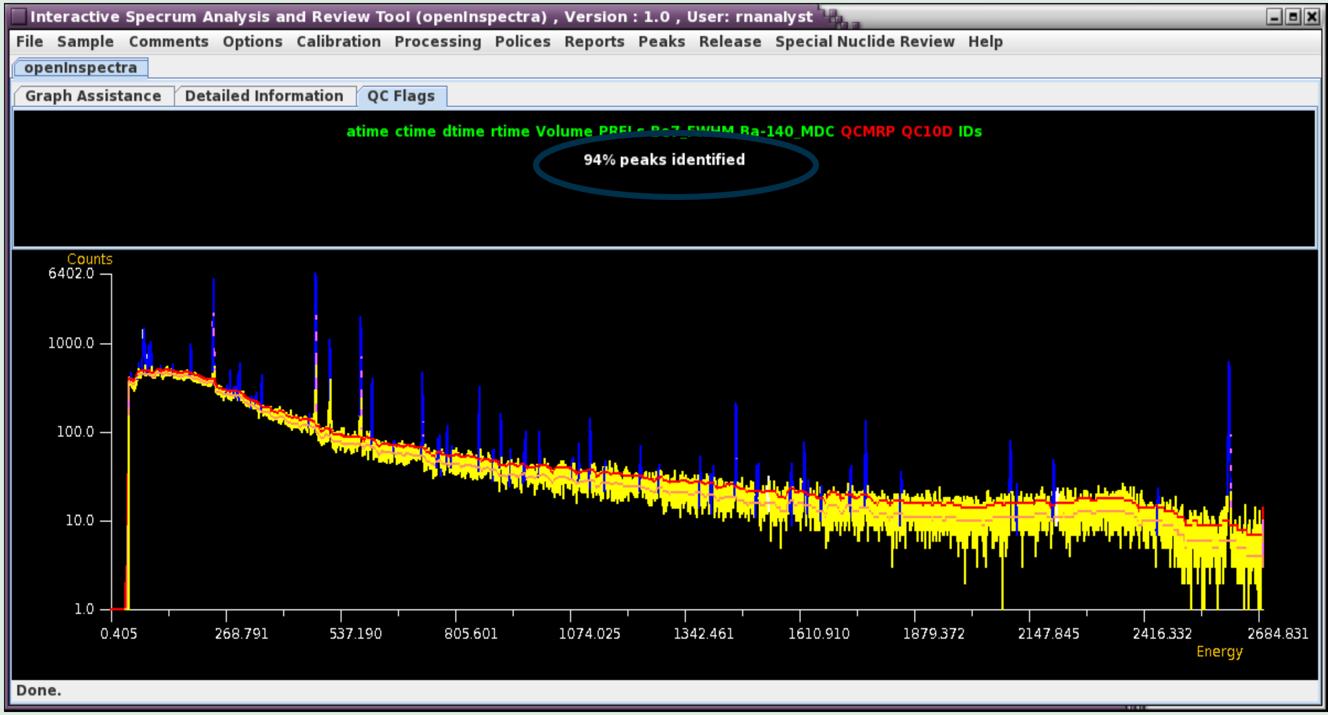
- A java-based license-free application for interactive review of automatic processing results which covers both particulates stations and Noble gas systems.

Sample metrics:

- Collection time
- Decay time
- Acquisition time
- Reporting time
- Be-7 FWHM
- Ba-140 MDC
- MRP QC
- QC 10 days
- %Identified peaks

Color coding:

- GREEN: *Pass*
- RED: *Fail*



Software for interactive review

- Some of peaks (value of energy) are sometimes not identified;
- One of main analyst tasks is to affect the correct nuclide to the corresponding value of energy. This task allows the precise identification and quantification of the elements;

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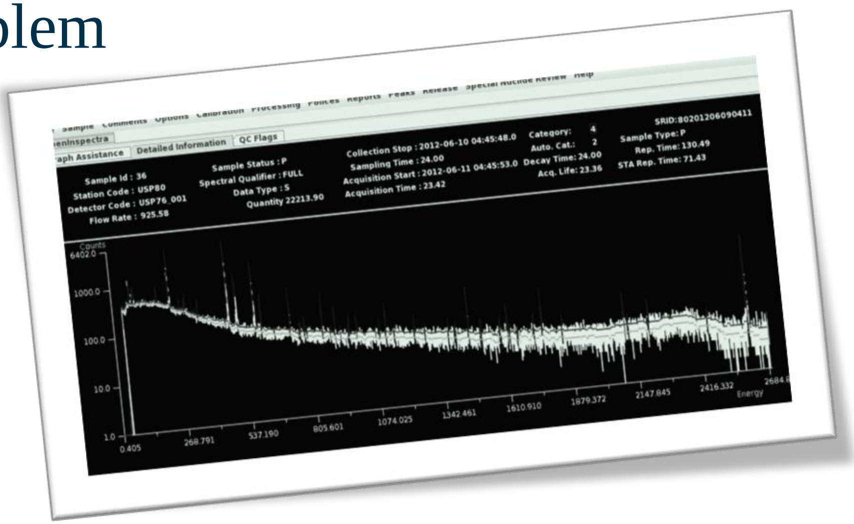

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- One of main analyst tasks is to affect the correct nuclide to the corresponding value of energy. This task allows the precise identification and quantification of the elements;
- Some of peaks which corresponding to a given value of energy are sometimes not identified;
- The non-identification is due to the fact that the corresponding energy does not correspond to any classified nuclide;
- In order to identify such a peak and associates it with one or more radionuclide, we apply a formulation of a subset sum problem.

- In fact, there are some peaks that are already associated to the corresponding radionuclide (automatic identification peak).
- These radionuclides along with **suspect radionuclides** are candidates to be the source of energy corresponding to the unidentified peak
- The problem can be expressed by the following :

From already detected radionuclides and the set of suspect radionuclides, what are the radionuclides whose energy accumulation corresponds to the energy of the unidentified peak.

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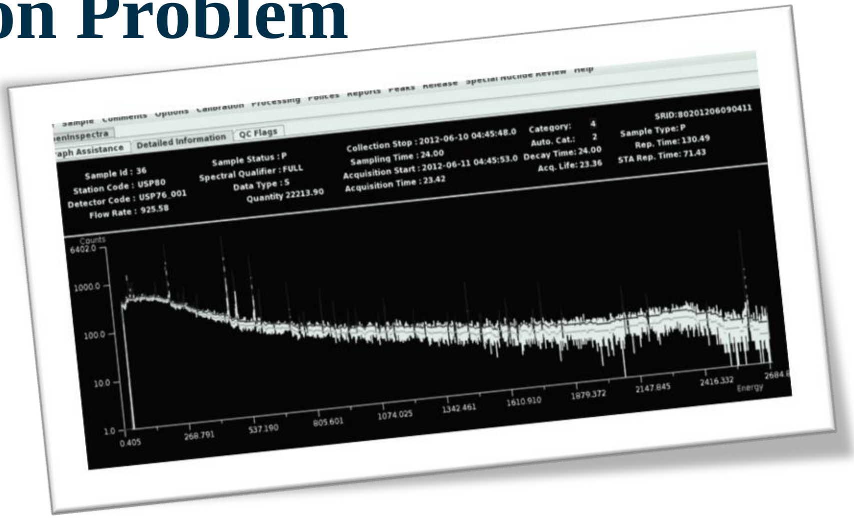

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- This problem can be formulated as a **functional problem** ***(A,s)*** of a **subset sum problem** as follows:

Input :

A : the vector which contain energy value of all existing radionuclides, together with energy value of suspect radionuclides

s : The energy of the unidentified peak (the peak which is not yet associated with a radionuclides)

Output :

The set of energy value selected (radionuclide detected).Then, the peak become identified

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- Let $A = (a_1, \ldots, a_n)$ a set of energy value of all existing radionuclides, together with energy value of suspect radionuclides
- Let $X = (X_1, \ldots, X_n)$ a set of boolean decision variable where

  $X_i = 1$ if the corresponding energy selected

  $X_i = 0$ *otherwise*
- Given s the energy of the unidentified peak

Identification of the peak consist on selecting a subset of X whose total corresponding energies is closest to, without exceeding, the value s

- Integer linear program ILP formulation

$$\text{Maximize } z = \sum a_i X_i$$

Subject to

$$\sum a_i X_i \leq s$$

$$X_i = 0 \text{ or } 1$$

Many approach resolution to the SSP problem

- Exact Algorithms *Faaland (1973); Ahrens and Finke(1975); Martello and Toth (1984)*
  - Dynamic programming,
  - Branch and bound method
- Approximate Algorithms
  - Greedy algorithms,
  - Polynomial time approximation schemes
  - fully polynomial time approximation schemes
  - Probabilistic analysis
- Hybrid Algorithms

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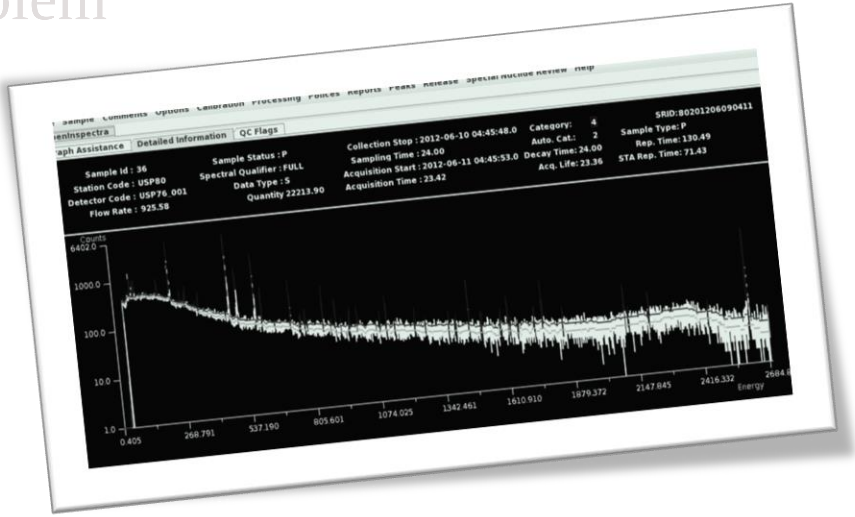

- SSP formulation for the problem of assignment of radionuclides to any peak without accordance energy-radionuclide;
- Our idea allows the analyst to reduce the number of unidentified peaks
- The SSP can be solved in polynomial or pseudo polynomial time for small instances
- The SSP formulation is very effective for testing the existence of such radionuclide which have several levels of energy

Thank you for your attention